

**Algebraic Topology: Old and New**  
**M. M. Postnikov Memorial Conference**

BĘDLEWO, 18-24.06.2007

ABSTRACTS

## PLENARY SPEAKERS AND TALKS

**Ralph Cohen**

### **Morse theory, Floer theory, and String Topology**

In this talk I will describe the use of Morse theory to study string topology. This involves the use of ribbon graphs and the study of moduli spaces of gradient graph flows in the loop space of a manifold. I will then discuss the relationship between Morse theory on the loop space of a manifold with Floer theory on its cotangent bundle. I will review results of geometers such as C. Viterbo, D. Salamon and J. Weber, and A. Abbondandolo and M. Schwarz on this topic, and then describe a homotopy theoretic aspect to this subject. Namely, we show that the Floer theory of the cotangent bundle can be “spectrified”, by showing that the spaces of J-holomorphic cylinders in the cotangent bundle are framed manifolds that represent, via Pontrjagin-Thom theory, the attaching maps of a spectrum. We then show that this spectrum is stably equivalent to the suspension spectrum of the free loop space.

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**Mikio Furuta**

### **The Pontrjagin-Thom construction and non-linear Fredholm theory (joint work with Tian-Jun Li)**

This is a report on a research in progress, which is a joint work with Tian-Jun Li. We generalize the classical Pontrjagin-Thom construction to construct a refined stable homotopy/framed bordism invariant for a family of non-linear Fredholm operators with fibrewise compact moduli spaces. Our theory is a mixture of homology and cohomology and admits a partial  $S$ -duality.

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**Soren Galatius**

### **Spaces of graphs and manifolds**

Pontryagin-Thom theory is usually applied to study smooth objects (manifolds). It played a prominent role in Madsen and Weiss’ proof of the “Mumford conjecture”. I will describe a framework for Pontrjagin-Thom theory for singular objects. Applying this for graphs we get the “Mumford conjecture for  $Aut(F_n)$ ”.

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**Stefan Haller**

**Complex valued Ray-Singer torsion  
(joint work with Dan Burghelea)**

The Ray-Singer torsion is a positive real number, defined with the help of zeta regularized determinants of self-adjoint Laplacians associated to a flat vector bundle over a closed Riemannian manifold. A theorem of Cheeger, Moeller and Bismut-Zhang asserts that, up to a well understood correction term, the Ray-Singer torsion coincides with the absolute value of the Reidemeister torsion - a non-vanishing complex number. In the talk we will discuss a complex valued Ray-Singer torsion, defined using zeta regularized determinants of non-selfadjoint Laplacians. Up to a computable correction term, this complex valued Ray-Singer torsion computes the Reidemeister torsion, including its phase. In full generality this was recently established by Su and Zhang. One nice feature of this complex valued Ray-Singer torsion is its holomorphic dependence on the flat connection.

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**Kaoru Ono**

**Floer cohomology and symplectic fixed points  
(joint work with Kenji Fukaya, Yong-Geun Oh and Hiroshi Ohta)**

I will report on the current status of our joint work with Kenji Fukaya, Yong-Geun Oh and Hiroshi Ohta concerning Floer theory for Lagrangian submanifolds. Using moduli spaces of holomorphic discs, we construct a certain algebraic object, what we call the filtered  $A_\infty$ -algebra associated to a Lagrangian submanifold. The obstruction to defining Floer cohomology can be formulated in terms of the Maurer-Cartan equation in this algebraic set-up. I would also like to mention some of applications.

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**Vladimir Sharko**

**Flows on manifolds and Bott functions**

*Definition.* The flow  $X_t$  on smooth closed manifold  $M^n$  belong to the class  $\Gamma(T^2)$  if the set of non-wandering points  $\Omega(X_t)$  of  $X_t$  consist of a disconnected union of embedded 2-tori with have normal hyperbolic structure.

*Theorem.* Let  $M^n$  be a smooth closed manifold and flow  $X_t$  belong to the class  $\Gamma(T^2)$ . Then exist  $T^2$ -Bott function  $f$  on  $M^n$ , such that singular set of  $f$  coincide with  $\Omega(X_t)$ .

The function  $f$  generates of the Kronrod-Reeb graph  $K - R(f)$ . We describe combinatorial conditions of the  $K - R(f)$ .

Denote by  $M_i^{T^2}(M^n)$  minimal number of 2-tori of index  $i$  from  $\Omega(X_t)$  taken

over all flows  $X_t$  on  $M^n$  from the class  $\Gamma(T^2)$ .

*Theorem.* Let  $M^n$  ( $n > 9$ ) be a smooth closed manifold such that  $\pi_1(M^n) = F_k$  - free group,  $\dim_{N[F_k]} H_2^j(M^n)$  - integer,  $\chi(M^n) = 0$ ,  $\pi_2(M^n) = 0$ . Then

$$M_i^{T^2}(M^n) = \dim_{N[F_k]} H_{(2)}^i(M^n) - \dim_{N[F_k]} H_{(2)}^{i-1}(M^n) + \dim_{N[F_k]} H_{(2)}^{i+2}(M^n) - \dots$$

We will discuss several results for symplectic manifolds with  $T^2$  action.

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**Stephan Stolz**

### **Super symmetric quantum field theories and generalized cohomology (Part 2)**

We first discuss how functional integration, over spaces of maps of a  $d$ -manifold to a target  $X$ , is related to integration over the fibre (or Gysin map) in the above cohomology theories. We then explain some of the results stated in Part 1 of the lecture.

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**Dennis Sullivan**

### **Applications of Algebraic Analogues of Postnikov Systems to Geometry**

Differential algebras of very general type admit treatment like manifolds or spaces subjected to methods of the homotopy category. The main algebraic notion to understand is homotopy between DGA maps from free DGAS to arbitrary algebras. Then free resolutions of general algebras, well defined up to homotopy, can be defined. An important technical feature for both of these points is that the differential in the free algebras is upper triangular for some free generating set. The structure of these free triangular DGAS can be understood in analogy to Postnikov systems.

The geometric applications are expressed in terms of homotopy classes of DGA maps from given free triangular objects into DGAS defined by algebraic topology and geometric topology. These DGA maps in some cases are themselves defined by regarding moduli spaces of solutions of certain PDEs as chains in the appropriate function spaces. The triangular property is equivalent to the celebrated bubbling and gluing theorems in these contexts e.g. antiself dual connections over four manifolds or  $J$  holomorphic curves in a symplectic manifold. In other cases like string topology or 3D invariants the DGA map is defined by more finite dimensional considerations like transversality and configuration space compactifications. The triangular property appears here as well.

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**Peter Teichner**

**Super symmetric quantum field theories and generalized  
cohomology (Part 1)**

We'll explain how to extend Graeme Segal's definition of a quantum field theory (QFT) over a manifold  $X$  in various directions, most significantly to super symmetric QFT's. We then discuss the resulting theories for  $d$ -dimensional space-time where  $d = 0, 1, 2$ . It turns out that  $d = 0$  leads to closed differential forms on  $X$ ,  $d = 1$  to geometric representatives for elements in  $K(X)$  and that a 2-dimensional susy QFT gives an integral modular function. Taking concordance classes of such QFT's over  $X$  leads to three basic (generalized) cohomology theories for  $d = 0, 1, 2$ : de Rham cohomology, K-theory and, conjecturally, TMF, the theory of topological modular forms.

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## HOMOTOPY THEORY SECTION

**Samer Al Ghour**

### **Homogeneity in Fuzzy Spaces and Their Induced Spaces**

If  $(X, T)$  is a homogeneous fuzzy topological space,  $a \in [0, 1)$  and  $Ta = \{f^{-1}(a, 1) : f \in T\}$ , then  $(X, Ta)$  is a homogeneized topological space. In this paper, we study the validity of the following implication depending on  $|X|$  and  $|T|$ .

$(X, Ta)$  is homogeneous for all  $a \in [0, 1)$  if and only if  $(X, T)$  is homogeneous. Some of our results answer two questions raised by Fora and Al Ghour in 2001.

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**Vigleik Angeltveit**

### **Blowing up the simplicial indexing category**

Sometimes one would like to construct a simplicial space, but one can only make the simplicial identities hold up to homotopy. I will explain how one can make an enriched version of the simplicial indexing category in such a way that a functor from this category is precisely a “simplicial” space where the simplicial identities hold up to homotopy (and higher homotopies). In fact, I will do this in two different ways. One makes the 2-sided bar construction on an  $A$ -infinity  $H$ -space into a generalized simplicial space, the other makes the cyclic bar construction into a generalized simplicial space. The construction uses the Stasheff associahedra. There is a notion of geometric realization of these generalized simplicial spaces, which uses the Stasheff associahedra in the first case and another family of polyhedra called cyclohedra in the second case.

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**Jonathan Ariel Barmak**

### **Simple homotopy types and finite spaces (joint work with Gabriel Minian)**

In this talk I will show recent results from a joint paper with G. Minian on simple homotopy types and finite spaces. We have found an elementary move in the setting of finite spaces which corresponds exactly to the concept of simplicial collapse of the associated simplicial complexes.

Recall that one can associate to each finite  $T_0$  space (poset)  $X$  the simplicial complex  $K(X)$  of nonempty chains of  $X$ . Conversely, one associates to a finite simplicial complex  $K$  the finite space  $X(K)$  of its simplices. I will prove that a collapse of finite spaces induces a simplicial collapse on their associated simplicial complexes and, on the other hand, if a simplicial complex  $K$  collapses simplicially to  $L$ , then the associated finite space  $X(K)$  collapses to  $X(L)$ .

The functors  $K$  and  $X$  induce a one to one correspondence between simple homotopy types of finite spaces and of finite simplicial complexes. We can then

use this new simple homotopy theory of finite spaces to study the classical one of CW-complexes, and on the other side, we use the classical machinery from CW-complexes to study finite spaces.

One of the most interesting thing about the elementary collapse of finite spaces is that it is much simpler to describe than the one of complexes because it consists on removing just one point of the space.

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**Sadi Bayramov**

### **Homotopic groups in fuzzy sets**

In this study, fuzzy homotopic groups are defined on the category of fixed pointed fuzzy sets. Exact sequence of homotopic groups is given. Further, we give a relationship between homotopic group of limit of inverse spectra of fuzzy sets and limit of inverse spectra of homotopic group of fuzzy sets.

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**Weimin Chen**

### **Differentiable Transformation Groups and Homotopy $K3$ Surfaces (joint work with Slawomir Kwasik)**

One of the basic problems in the theory of differentiable transformation groups is to understand how the groups may depend on the underlying differentiable structure. Such questions have been extensively studied in the past, especially for the case of exotic spheres of dimension greater than or equal to 7. On the other hand, differentiable transformation groups of exotic 4-manifolds have remained to be a largely untest territory. In this talk, we will discuss some recent progress in this direction.

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**Hellen Colman**

### **Lusternik-Schnirelmann theory for orbifolds as Lie groupoids**

We propose a new numerical invariant for Lie groupoids which generalizes the Lusternik-Schnirelmann category of topological spaces. This number is invariant under Morita equivalence, then yields a well defined  $LS$ -category for orbifolds. An orbifold map is given by an equivalence class of generalized maps between Lie groupoids. These generalized maps are obtained by formally inverting essential equivalences. We develop a notion of Morita homotopy between generalized maps and prove that the  $LS$ -category of a Lie groupoid is a homotopy invariant. We describe a bicategory of fractions where our notion of Morita homotopy equivalence amounts to isomorphism of objects and defines the orbifold homotopy type.

In the homotopical side of the subject, the  $LS$ -category of an orbifold relates to the nilpotencies of various algebraic objects associated to its groupoid presentation. In the more analytical side, we show that the  $LS$ -category provides a lower bound for the number of critical points of any smooth function on a compact orbifold.

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**Jose Manuel Garcia Calcines**

### **A Whitehead-Ganea approach for proper Lusternik-Schnirelmann category**

Proper homotopy invariants of Lusternik-Schnirelmann type were successfully introduced by R. Ayala, E. Dominguez, A. Marquez and A. Quintero in 1992. It is known that the proper category is an  $I$ -category in the sense of Baues, so only a limited set of homotopy categorical construction are available. For example, homotopy limits and fibrations do not exist in the proper setting. This prevents the classical (Ganea's and Whitehead's) axiomatic formulations of  $LS$ -invariants, which constitutes a serious obstacle for the development of these invariants in proper homotopy theory.

In this work we establish Whitehead and Ganea characterizations for proper  $LS$ -category by considering the category of exterior spaces. This category is complete and cocomplete and contains the proper category through a full embedding. Furthermore it has a suitable closed model structure of Strom type. Then, from the axiomatic  $LS$ -category arising from the exterior homotopy category we can recover the corresponding  $LS$ -invariants.

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**Cigdem Gunduz Aras**

### **Homotopy Theory on the Category of Fuzzy Topological Spaces**

In this study, homotopic theory is constructed on the category of fixed pointed fuzzy topological spaces. Homotopic exact sequence is given for fuzzy fibration. Further, we give a relationship between homotopic group of limit of inverse spectra of fuzzy topological space and limit of inverse spectra of homotopic group of fuzzy topological spaces.

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**Andre Henriques**

**Higher Clifford algebras  
(joint work with Arthur Bartels and Chris Douglas)**

Clifford algebras and topological  $K$ -theory are intimately related: one can describe the higher  $K$ -theory groups via bundles of  $Cliff(n)$ -modules, and the Morita equivalence between  $Cliff(n)$  and  $Cliff(n2)$  implies Bott periodicity. The Clifford algebras are best viewed as objects in the symmetric monoidal 2-category of  $Z/2$ -graded algebras and bimodules. For example, one can then characterize them as the invertible objects in that 2-category. Let  $vN2$  be the symmetric monoidal 2-category of von Neuman algebras and bimodules, composition being Connes fusion. Stolz and Teichner conjectured the existence of a symmetric monoidal 3-category  $C$  with the property that  $Hom_C(1, 1) = vN2$ . The invertible objects in that 3-category category should play the same role for Elliptic cohomology than Clifford algebras do in  $K$ -theory. Namely, the group of invertible objects should be a small abelian group that is somehow related to the periodicity of Elliptic cohomology. Conformal nets are the objects of our candidate symmetric monoidal 3-category. We also have a candidate for the generator of the group of invertible objects: the net of local fermions. We don't know yet whether the net of local fermions is of finite order or not, but we do have an argument that shows that it's at least of order 24.

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**Tornike Kadeishvili**

**$A(\infty)$ -algebra structure in cohomology and rational homotopy type**

It is obvious that the rational cohomology algebra  $H^*(X, Q)$  do not define the rational homotopy type. We show that this cohomology is naturally equipped with an additional structure, namely with a structure of commutative  $A(\infty)$ -algebra  $(H^*(X, Q), \{m_i : H^*(X, Q)^i \rightarrow H^*(X, Q), i = 2, 3, \dots\})$  and this object completely determines the rational homotopy type. We present various applications of this complete rational homotopy invariant. The roots of this work go to my 1979 talk on M. Postnikov's seminar.

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**Ralf Kaufmann**

**Homotopy Theory and Moduli Spaces**

Recently there has been a spur of activity on operadic actions on the Hochschild co-chain complex wich can be traced back to three sources: string topology, the various versions of Deligne's conjecture and the mathematical aspects of  $D$ -branes from string theory. We will discuss how all of these aspects can be unified in a moduli space setting. This has the two-fold advantage of both making the known constructions appear naturally as well as providing a vast generalization of them.

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**Akeh Mac Henry**

### **Constructing and Characterizing Degree $n$ Functors**

Let  $F$  be a functor from a basepointed category with finite coproducts to a category of chain complexes over an abelian category. Such a functor is homologically degree  $n$  if its  $(n - 1)$ -st cross effect (in the sense of Eilenberg and Mac Lane) has trivial homology. We describe a method for constructing, by means of cotriples associated to the cross effects of  $F$ , a universal tower,

$$\dots \rightarrow P_{n1}F \rightarrow P_nF \rightarrow \dots \rightarrow P_1F \rightarrow P_0F = F(*)$$

in which each functor  $P_nF$  is homologically degree  $n$ . This construction arose from the study of Goodwillie's Taylor tower in the case of functors of modules over a ring. Using this model, we will characterize homologically degree  $n$  functors in terms of modules over a certain DGA, and discuss some related constructions due to Eilenberg-Mac Lane, and Dold-Puppe.

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**Vassily Olegovich Manturov**

### **Khovanov homology: Topology and Combinatorics**

We describe various aspects of Khovanov homology theory and its generalisations for a larger class of objects rather than usual classical knots (virtual knots, twisted virtual knots, etc). On one hand, this leads to certain difficulties in the original definition which should be rewritten. On the other hand, additional topological or combinatorial structures allow one to find additional gradings and filtrations which lead to spectral sequences converging to the "original" Khovanov homology. These theories turn out to be projectively functorial thus leading to an extension of Rasmussen's invariant.

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**Elke Markert**

### **A model for (connective) $K$ -theory derived from field theories**

We describe an  $\Omega$ -spectrum for connective  $ko$ -theory formed from spaces  $inf_n$  of operators which have certain nice spectral properties, and which fulfill a connectivity condition. Dropping the connectivity condition we obtain operator spaces  $Inf_n$ : These are homotopy equivalent to the spaces  $EFT_n$  of 1-dimensional supersymmetric euclidean field theories of degree  $n$  which were defined by S. Stolz and P. Teichner (in terms of certain homomorphisms of super semi-groups). They showed that the  $EFT_{-n}$  are homotopy equivalent to  $KO_n$ . We can derive a homotopy equivalent version of the connective  $\Omega$ -spectrum  $inf$  in terms of "field theory type" super semi-group homomorphisms,  $eft$ .

**Dariusz Miklaszewski**

**The Conjecture on the Brouwer Fixed Point Theorem for  
 $1 - S^k$ -mappings**

The  $1 - S^k$ -mapping of  $D^n$  is a set-valued mapping of the  $n$ -dimensional disc, which is continuous with respect to the Borsuk metric of continuity and takes values homeomorphic to either a point or the  $k$ -sphere. We know that The Brouwer Fixed Point Theorem for such mappings holds for  $k = 0, n - 1, n - 2$ . We ask if  $1 - S^1$ -mappings of  $D^4$  have fixed points.

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**Dmitry Millionschikov**

**Massey products in Lie algebra cohomology**

We discuss Massey products in a positively graded Lie algebra cohomology. One of the main examples is so-called “positive part”  $L_1$  of the Witt algebra  $W$ . Buchstaber conjectured that  $H^*(L_1)$  is generated with respect to non-trivial Massey products by  $H^1(L_1)$ . Feigin, Fuchs and Retakh represented  $H^*(L_1)$  by trivial Massey products and the second part of the Buchstaber conjecture is still open. We consider an associated graded algebra  $\mathfrak{m}_0$  of  $L_1$  with respect to the filtration by the its descending central series and prove that  $H^*(\mathfrak{m}_0)$  is generated with respect to non-trivial Massey products by  $H^1(\mathfrak{m}_0)$ .

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**Elias Gabriel Minian**

**Massey products in Lie algebra cohomology**

In this talk I will explain the main results on homotopy theory of finite spaces and its relationship with homotopy theory of (general) spaces. In the first part of the talk I will recall results of P. S. Alexandroff, M. McCord and R. Stong on the relationship between finite spaces, posets and polyhedra and on the homotopy types of finite spaces from a combinatorial point of view. In the second part of the talk I will show new results from a joint work with J. A. Barmak concerning the minimal finite models of spaces. A minimal finite model of a topological space  $X$  is a space with the minimum number of points which is weak equivalent to  $X$ . I will exhibit the minimal finite models of the spheres and of finite graphs and show new techniques based on finite spaces to compute homotopy invariants of general spaces using the combinatorics and the topology of these objects.

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## Cornel-Sebastian Pinte

### Mappings with high-dimensional critical sets

We evaluate, in terms of dimension, the critical sets of certain mappings. We particularly focus our attention on mappings of zero degree, as well as, on mappings whose homotopy classes are equally represented by maps with only critical points.

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## Leonid Plachta

### Remarks on invariants of finite order of links in a 3-space: geometric aspects

We show that for every knot  $K$  with genus  $g(K)$  and any  $n \in \mathbf{N}$  and  $m \geq g(K)$  there exists a prime knot  $L$  which is  $n$ -equivalent to  $K$  and has the genus  $g(L)$  equal to  $m$ . We also discuss relationship between Vassiliev invariants and the canonical genus of a knot.

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## Semen Podkorytov

### On homotopy invariants of maps to the circle

For a homotopy invariant  $f : [X, Y] \rightarrow L$  ( $L$  is an abelian group) we define its *order*:  $\text{ord } f \leq r$  if for a map  $a : X \rightarrow Y$  the value  $f([a])$  depends  $\mathbf{Z}$ -linearly on the characteristic function  $I_r(a) : (X \times Y)^r \rightarrow \mathbf{Z}$  of the  $r$ th Cartesian power of the graph of  $a$ .

For  $Y = S^1$ , we determine  $\text{ord } f$  for arbitrary  $f$ . As a corollary, we obtain a lower bound of the order of the identity map  $[S^{n_k}, S^k] \rightarrow \pi_n^s$  for  $n \leq 3$  and large  $k$ .

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## Paolo Salvatore

### Cacti operads and the topological Deligne conjecture

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## Vladimir Smirnov

### Bott periodic theorem and differentials of the Adams spectral sequence

Bott periodic theorem will be used to obtain formulas for higher differentials of the Adams spectral sequence of stable homotopy groups of spheres.

**Ross Staffeldt**

**The connecting homomorphism for  $K$ -theory of generalized free products**

For a generalized free product situation where  $R$  is the generalized free product of rings  $B$  and  $C$  over a common subring  $A$  the connecting homomorphism from the  $K$ -theory of  $R$  to the  $K$ -theory of  $A$  and the Nil-term is of interest. I will describe how to use techniques from Waldhausen's approach to algebraic  $K$ -theory to develop a framework in which it is possible to describe this connecting homomorphism explicitly.

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**Thomas Tradler**

**String operations in an algebraic setting**

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**Nathalie Wahl**

**Stabilizing mapping class groups of 3-manifolds  
(joint work with Allen Hatcher)**

Let  $M$  be a compact, connected 3-manifold with a fixed boundary component  $d_0M$ . For each prime manifold  $P$ , we consider the mapping class group of the manifold  $M_n^P$  obtained from  $M$  by taking a connected sum with  $n$  copies of  $P$ . We prove that the  $i$ th homology of this mapping class group is independent of  $n$  in the range  $n > 2i + 1$ . Our theorem moreover applies to certain subgroups of the mapping class group and include, as special cases, homological stability for the automorphism groups of free groups and of other free products, for the symmetric groups and for wreath products with symmetric groups.

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**Scott Wilson**

**Combinatorial Topology, Homotopy Algebra and Fluids  
(joint work with Dennis Sullivan)**

We'll describe how a homotopy algebra appearing on the cellular cochains of a cell-decomposed manifold can be used to give combinatorial models for fluid flow. This approach using algebraic topology gives computable models at various levels of scale or subdivision of the space.

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**Mahmoud Zeinalian**

**Closed TCFT for Calabi-Yau elliptic spaces  
(joint work with Kevin Costello and Thomas Tradler)**

We give an explicit action of the moduli space of Riemann surfaces (with closed boundaries), at the chain level, on the Hochschild complex of a Hermitian Calabi-Yau elliptic space. Among applications are extension of the String Topology operations to all chains on the moduli space of Riemann surfaces, as well as a more explicit version of Costello's  $B$ -model at all genera.

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## MANIFOLDS TOPOLOGY SECTION

**Petr Akhmetiev**

### **Geometric approach towards stable homotopy groups of spheres I, Adams-Hopf invariants**

In the paper a geometric approach toward stable homotopy groups of spheres, based on Pontrjagin-Thom construction is proposed. From this approach a new proof of Hopf Invariant One Theorem by J. F. Adams for all dimensions except 15, 31, 63, 127 is obtained. It is proved that for  $n > 127$  in stable homotopy group of spheres  $\Pi_n$  there is no elements with Hopf invariant one. The new proof is based on geometric topology methods. The Pontrjagin-Thom theorem (in the form proposed by R. Wells) about the representation of homotopy groups of the real projective infinite-dimensional space (this groups is mapped onto 2-components of stable homotopy groups of spheres by the Khan-Priddy theorem) by cobordism classes of immersions of codimension 1 of closed manifolds (generally speaking, non-oriented) is considered. The Hopf Invariant is expressed as a characteristic classes of the dihedral group for the self-intersection manifold of an immersed codimension 1 manifold that represents the given element in the stable homotopy group. In the new proof a Geometric Control Principle (by M. Gromov) for immersions in the given regular homotopy classes based on the Smale-Hirsh Immersion Theorem is required.

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**Alexey Chernavsky**

### **About local contractible neighborhoods in manifold homeomorphism groups**

There will be outlined an approach to a direct proof of the existence of local contractible neighborhoods in manifold homeomorphism groups (that is without resorting to the help of pl topology).

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**Vladimir Chernov**

### **Causality and Linking in Globally Hyperbolic Spacetimes (join work with Yuli Rudyak)**

We construct a generalization *alk* of the linking number to the case of nonzero homologous linked submanifolds and apply it to the study of causality in Lorentzian manifolds. Let  $M^m$  be a spacelike Cauchy surface in a globally hyperbolic spacetime  $(X^{m+1}, g)$ . The spherical cotangent bundle  $ST^*M$  is identified with the space  $N$  of all null geodesics in  $(X, g)$ . Hence the set of null geodesics passing through a point  $x \in X$  gives an embedded  $(m - 1)$ -sphere  $S_x$  in

$N = ST^*M$  called the sky of  $x$ . Low observed that if the link  $(S_x, S_y)$  is nontrivial, then  $x, y \in X$  are causally related. The spheres  $S_x$  are isotopic to fibers of  $(ST^*M)^{2m-1} \rightarrow M^m$ . They are nonzero homologous and  $lk(S_x, S_y)$  is undefined when  $M$  is closed, while  $alk(S_x, S_y)$  is well defined. Moreover,  $alk(S_x, S_y) \in Z$  if  $M$  is not an odd-dimensional rational homology sphere. If  $(X, g)$  has all the timelike sectional curvatures nonnegative and  $alk$  takes values in  $Z$ , then  $x, y \in X$  are causally related if and only if  $alk(S_x, S_y) \neq 0$ . We prove that if  $alk$  takes values in  $Z$  and  $y$  is in the future of  $x$ , then  $alk(S_x, S_y)$  equals to the intersection number of any future directed past inextendible timelike curve to  $y$  and of the null cone of  $x$ . We show that  $x, y$  in nonrefocussing  $(X, g)$  are causally unrelated iff  $(S_x, S_y)$  can be deformed to a pair of  $S^{m-1}$ -fibers of  $ST^*M \rightarrow M$  by an isotopy through skies.

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**Alexander Felshtyn**

### **New directions in the Nielsen-Reidemeister theory**

1. Nielsen theory and Symplectic Geometry. Nielsen numbers as dimensions of symplectic Floer homology for symplectomorphisms of surface.
2. Reidemeister theory and Noncommutative Geometry. Twisted Burnside-Frobenius theorem for automorphisms of residually finite groups, dynamics on the unitary dual space and congruences for Reidemeister numbers of iterations.
3. Twisted conjugacy classes and Geometric Group theory. Reidemeister numbers are infinite for automorphisms of the Gromov hyperbolic groups and Baumslag-Solitar groups.

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**Bernhard Hanke**

### **Positive scalar curvature with symmetry**

The existence question for metrics of positive scalar curvature is discussed in an equivariant context. In particular, we describe a new codimension - 2 surgery technique which removes singular strata from fixed point free  $S^1$ -manifolds while preserving equivariant positive scalar curvature. This is applied to derive the following theorem: each closed fixed point free  $S^1$ -manifold of dimension at least 6 whose isotropy groups have odd order and whose union of maximal orbits is simply connected and not spin, carries an  $S^1$ -invariant metric of positive scalar curvature.

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**Norio Iwase**

**Relative  $L - S$  category and categorical length**

Three different definitions of relative  $L - S$  categories are currently known: one by Berstein and Ganea, one by Fadell and Husseini and one by Arkowitz and Lupton. We give new relative  $L - S$  category to unify these definitions, which is also used to give a definition of the categorical sequence in the sense of Fox. Using it, we show some results on the original  $L - S$  category which is known as the results on strong category.

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**Uwe Kaiser**

**Khovanov theory on oriented surfaces**

We discuss Bar-Natan's geometric complex for diagrams on surfaces  $F$ . This leads to the problem of understanding a bordism category with objects given by (essential) curve systems on  $F$ , and bordisms given by properly embedded surfaces in  $F \times I$ , quotiented out by Bar-Natans local relations. In general the bordisms will be nonorientable. We discuss special cases where the nonorientability can be avoided, e.g. for diagrams, which are trivial in  $Z/2Z$ -homology. We also discuss the problems of constructing TQFT's in the general situation.

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**Maxim Kazarian**

**Enumeration of complex nodal curves by the methods of cobordism theory**

We consider the problem of counting complex curves on the plane having a given degree, a given number of double points and passing through a given collection of points, and a similar problem for a general linear system of curves on arbitrary complex surface. Kleiman and Piene made explicit computations of these numbers by algebraic geometry methods of intersection theory in the case when the number of double points does not exceed 8. Based on the ideas of cobordism theory, we develop a method that extends the computations of Kleiman and Piene to at least 20 of singular points. Our method is rather elementary and does not require the detailed study of the geometry of singular varieties related to the problem. The input data used in this method is just a combinatorial information such that the quasihomogeneity degrees of singular germs of small codimensions. At the moment, the method has no complete justification and the computations made on its basis are only conjectural. However, there are many evidences ensuring the validity of the conjecture.

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**Ulrich Koschorke**

**Coincidence manifolds and homotopy theory**

In topological fixed point theory one attempts to minimize the number of fixed points of a selfmap  $f$  of a compact  $n$ -dimensional manifold by varying  $f$  within its homotopy class. A lower bound (which is sharp except possibly when  $n = 2$ ) was described by the Danish mathematician Jakob Nielsen. In our talk we extend Nielsen theory to coincidences of pairs of maps between manifolds of arbitrary (and, in general, different) dimensions. Our approach is very geometric and involves path spaces as well as nonstabilized normal bordisms. It turns out to be intimately related to deep notions of homotopy theory such as Kervaire invariants and various versions of Hopf invariants (of e.g. Ganea, Hilton or James).

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**Sergiy Maksymenko**

**Fundamental groups of orbits of Morse functions on surfaces**

Let  $M$  be a compact surface,  $P$  be either a real line or a circle, and  $f : M \rightarrow P$  be a Morse mapping. Denote by  $O_f$  the right orbit of  $f$ , i.e. the orbit of  $f$  with respect to the action of the group of diffeomorphisms  $Diff(M)$  of  $M$ .

(1) We show that  $O_f$  is homotopy equivalent to some covering space of the  $n$ -th configuration space of  $M$ , where  $n$  is a total number of critical points of  $f$ .

It was proved by the author earlier, that if  $M$  is of genus  $g \geq 2$ , then  $O_f$  is aspherical. This implies that  $\pi_1 O_f$  is a subgroup of  $n$ -th braid group of  $M$ .

(2) We also construct a finite presentation for the group  $\pi_1 O_f$  for the case  $g \geq 2$ . It turns out that this presentation has similarity with the presentation for Artin groups. In particular, analogues of  $K(\pi, 1)$  conjecture and Tits conjecture on squares of generators in Artin group can easily be formulated for  $\pi_1 O_f$ .

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**Volker Puppe**

**Involutions on 3-Manifolds and Self-dual, Binary Codes  
(joint work with M. Kreck)**

An involution on a compact 3-manifold with only isolated fixed points determines a self-dual, binary code and every such code can be obtained from an involution. Doubly-even codes are related to Spin-manifolds.

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**Michael Sullivan**

**String topology and contact homology  
(joint work with Dennis Sullivan)**

I will discuss some work trying to connect the open string topology of a manifold with the relative contact homology (a theory based on pseudo-holomorphic curves) of its Legendrian unit conormal bundle. No prior knowledge of symplectic or contact geometry will be assumed. This is motivated by work of Cieliebak and Latschev.

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**Andreas Szücs**

**Classifying spaces in singularity theory**

The relationship of two classifying spaces is established. The first one was invented by Kazarian, the second by the author. The relationship was conjectured by Kazarian. This relationship has many very concrete consequences concerning the cobordism classification of singular maps.

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## LOW DIMENSIONAL TOPOLOGY SECTION

**Boris Apanasov**

### **Reflection groups in 3-sphere and hyperbolic 4-cobordisms**

We answer the question whether a reflection group  $G$  in 3-sphere with its fundamental 3-polyherdon which consists of two connected components homeomorphic to 3-ball and having the same combinatorial (hyperbolic) type is a quasiconformal conjugation of a Fuchsian group. In other words whether the corresponding quotion 4-orbifold is a trivial hyperbolic cobordism with compact boundary (hyperbolic) components.

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**Khaled Bataineh**

### **Vassiliev Invariants for Knots and Links in the Solid Torus**

We define a type-one invariant for links with zero winding number in the solid torus. We show that this invariant is universal. Moreover, we show that this invariant is a generalization of Aicardi's invariant for knots and links in the solid torus.

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**Stefan Bauer**

### **On refined Seiberg-Witten theory**

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**Cynthia Curtis**

### **Vanishing of $SL_2(\mathbb{C})$ Casson invariants (joint work with Hans U. Boden)**

We discuss the  $SL(2, \mathbb{C})$  Casson invariant and its properties. We give conditions which ensure that the invariant is nonvanishing. We then use the splice construction to find an infinite family of 3-manifolds for which the invariant vanishes.

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**Stefan Friedl**

**Symplectic manifolds of the form  $S^1 \times N^3$   
(joint work with Stefano Vidussi)**

Thurston showed that if a 3-manifold  $N$  fibers over  $S^1$ , then  $S^1 \times N^3$  is symplectic. It is an open question whether the converse holds. In this talk I will talk about joint work with Stefano Vidussi on this question. In particular we give an affirmative question for several special cases and we reduce this question to a question on separability of subgroups in 3-manifolds.

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**Kim Froyshov**

**Floer homology and 4-manifolds with  $b_1 = 1$  and  $b^+ = 0$**

We explain how monopole Floer homology may be used to define invariants of certain closed, oriented smooth 4-manifolds with  $b_1 = 1$  and  $b^+ = 0$ .

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**Hiroshi Goda**

**Circle-valued Morse maps, Reidemeister torsions, and sutured  
manifold theory  
(joint work with Hiroshi Matsuda and Andrei Pajitnov)**

We discuss on an extension of the works of Hutchings-Lee and Jiang-Wang. Further, we give concrete examples of the calculations of the related Reidemeister torsions and zeta functions using the sutured manifold theory.

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**Takayuki Morifuji**

**On a secondary invariant of the hyperelliptic mapping class group**

In this talk, we discuss various aspects of Meyer's function of the hyperelliptic mapping class group of a surface, which is defined as a secondary invariant of the signature cocycle.

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**Andreas Nemethi**

**Surgery formula for the Seiberg-Witten invariants**

We present a cut-and-paste surgery formula of Seiberg-Witten invariants for negative definite plumbed rational homology 3-spheres. The proof is based on a new surgery formula for the Reidemeister-Turaev torsion. It is similar to (and motivated by) Okuma's recursion formula targeting analytic invariants of splice quotient surface complex singularities (introduced by Neumann and Wahl). The two formulas combined provide automatically a proof of the equivariant version of the 'Seiberg-Witten invariant conjecture' for these singularities. (This conjecture related the analytic invariant, e.g. the geometric genus of some complex surface singularities with the Seiberg-Witten invariants of their link, it generalizes the 'Casson invariant conjecture' of Neumann and Wahl, valid for complete intersections.)

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**Hiroshi Ohta**

**Symplectic fillings and normal singularities  
(joint work with Kaoru Ono)**

I will talk about some results on rigidity/flexibility of symplectic fillings of links of normal surface singularities. This is based on joint works with Kaoru Ono.

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**Nikolai Saveliev**

**Rohlin's invariant and gauge theory  
(joint work with Daniel Ruberman)**

This is an ongoing project with Daniel Ruberman about the relationship between the classical Rohlin invariant and certain invariants coming from 4-dimensional gauge theory (both Yang-Mills and Seiberg-Witten). The interaction between these two types of invariants provides insight into some old problems concerning the homology cobordism group of homology 3-spheres, as well into existence of metrics of positive scalar curvature on certain non-orientable manifolds.

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**Nadya Shirokova**

**On classification of Floer-type theories**

Many Floer-type theories can be considered as categorifications of classical invariants. We propose a program of classification of such theories. We construct Floer-type local systems on the spaces of manifolds (including singular ones), extend them to the singular locus and introduce the definition of a local system of finite type. Our main examples come from Khovanov and Ozsvath-Szabo theories.

**Daniel Silver**

**Dynamics of Twisted Alexander invariants**

Twisted Alexander invariants were introduced by X. S. Lin and later extended by Wada, Kirk, Livingston and others. We examine twisted Alexander modules of knots and links from the perspective of algebraic dynamics. For any nontrivial knot  $k$ , we show that there exists a finite representation of its group such that the associated twisted Alexander polynomial is nontrivial. A parabolic representation of a knot or link group determines an algebraic dynamical system; we give a homological interpretation of its topological entropy.

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**Andrei Teleman**

**Instanton and curves on non-Kählerian surfaces**

The classification of complex surfaces of Kodaira class VII is not understood yet. This is the most important gap in the Enriques-Kodaira classification table. Using methods from Donaldson theory, we prove that any minimal class VII surface with second Betti number  $b_2 = 1$  or  $b_2 = 2$  has a cycle of curves. This result completes the classification of these surfaces, up to deformation equivalence. In particular, it implies that any such surface is diffeomorphic to a blown up primary Hopf surface (in one or two points). For  $b_2 = 1$  one obtains a complete classification, up to biholomorphic isomorphisms.

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**Masaki Ue**

**The Fukumoto-Furuta and the Ozsvath-Szabo invariants for spherical 3-manifolds**

The Fukumoto-Furuta invariant is defined for a 3-manifold with spin structure bounded by a spin 4-orbifold and is identified with the Neumann-Siebenmann invariant in case of a Seifert rational homology 3-sphere. We show that for a spherical 3-manifold this invariant is essentially the same as the Ozsvath-Szabo invariant, although they are quite different in general.

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**Susan Williams**

**Algebraic Dynamcis of Knots and Links**  
(joint work with Daniel Silver)

We present joint work with Daniel Silver on applications of the theory of algebraic dynamical systems to knots and links. We study the Alexander module of a link of  $d$  components via its Pontrjagin dual, a compact abelian group with an action by  $d$  commuting automorphisms. Periodic points of this  $Z^d$  dynamical system correspond to elements of the homology of finite abelian covers of  $S^3$  branched over the link. We use a theorem of Lind, Schmidt and Ward on periodic points and entropy of algebraic dynamical systems to interpret the logarithmic Mahler measure of the Alexander polynomial as a growth rate of the orders of torsion subgroups of these homology groups.

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## MANIFOLDS TOPOLOGY II

**Malkhaz Bakuradze**

### **Transferred Chern classes and Morava $K$ -theory rings**

Morava  $K$ -theory rings of classifying spaces for various finite groups are calculated in terms of transferred Chern characteristic classes.

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**Dmitry Bolotov**

### **Macroscopic dimension of Manifolds**

We construct a closed manifold having torsion-free fundamental group and whose universal covering is of macroscopic dimension 3. This yields a counterexample to Gromov's conjecture about the falling of macroscopic dimension. Early we have proved, that Gromov's conjecture is true in dimension 3.

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**Alexander Gaifullin**

### **Combinatorial formulae for the Pontrjagin classes**

The problem of finding combinatorial formulae for the Pontrjagin classes of triangulated manifolds originated from the pioneer work by I. M. Gelfand, A. M. Gabrielov, and M. V. Losik in 1975. The problem is posed as follows. Given a combinatorial manifold construct explicitly a simplicial cycle whose homology class is the Poincaré dual of a given polynomial in the Pontrjagin classes of the manifold. Since then different authors (I. M. Gelfand, A. M. Gabrielov, M. V. Losik, R. MacPherson, J. Cheeger) constructed several different formulae. Unfortunately, each of these formulae either involves some non-combinatorial data (for instance, a smoothing) or is not computable by a combinatorial algorithm. The first directly computable formula for the first Pontrjagin class was constructed by the author in 2004. The coefficient of each simplex in the cycle obtained by this formula is determined solely by the combinatorial structure of the link of the simplex. It appears that these coefficients necessarily have unbounded denominators. We also describe how to obtain algorithmically computable combinatorial formulae for the first Pontrjagin class with bounded denominators. The method is based on the theory of bistellar moves.

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**Mark Grant**

**Sectional category weight and topological complexity**

We introduce the notion of sectional category weight of a cohomology class. This generalises the usual category weight (introduced by Fadell and Husseini and developed by Rudyak), and may be used to improve the classical cohomological lower bound for the sectional category (or Schwarz genus) of a map.

We apply these ideas to the computation of topological complexity - a numerical homotopy invariant of interest to roboticists, defined by Farber to be the sectional category of the free path fibration. In particular, we show how elements of high category weight lead to elements of high sectional category weight with respect to the free path fibration. As a nice application we compute the topological complexity of the Borromean rings link complement.

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**Friedrich Hegenbarth**

**Applications of controlled surgery in dimension four**

There will be given examples of four-dimensional surgery problems which can be solved by use of the controlled theory. Among these examples are surgeries of degree-1 normal maps between closed four-manifolds with free non-abelian fundamental groups.

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**Giorgi Khimshiashvili**

**Holomorphic structures in Seifert fibrations**

We will deal with holomorphic curves in the unparameterized loop space of orientable closed three-dimensional manifold endowed with a natural almost complex structure introduced by J. L. Brylinski. It will be shown that links of isolated singularities of two-dimensional complete intersections provide examples of holomorphic curves in loop spaces. In particular, the inverse map to Hopf fibration considered as a map from the Riemann sphere to the Brylinski loop spaces of three-dimensional sphere with the round metric appears to be holomorphic. It will be also shown that, for a wide class of Seifert fibrations, the natural map from the factor-space to the loop space becomes holomorphic after a suitable leafwise diffeomorphism. Further visual examples will be constructed using deformations of isolated plane curve singularities.

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**Ulrich Koschorke**

**Coincidence manifolds and homotopy theory**

In topological fixed point theory one attempts to minimize the number of fixed points of a selfmap  $f$  of a compact  $n$ -dimensional manifold by varying  $f$  within its homotopy class. A lower bound (which is sharp except possibly when  $n = 2$ ) was described by the Danish mathematician Jakob Nielsen.

In our talk we extend Nielsen theory to coincidences of pairs of maps between manifolds of arbitrary (and, in general, different) dimensions. Our approach is very geometric and involves path spaces as well as nonstabilized normal bordisms. It turns out to be intimately related to deep notions of homotopy theory such as Kervaire invariants and various versions of Hopf invariants (of e.g. Ganea, Hilton or James.).

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**Andrzej Krzysztof Kwasniewski**

**On extended finite operator calculus as an example of algebraization of analysis**

“A Calculus of Sequences” started in 1936 by Ward constitutes the general scheme for extensions of classical operator calculus of Rota Mullin considered by many afterwards and after Ward.

This calculus is an example of the algebraization of the analysis, here restricted to the algebra of formal series in general.

We make explicit umbral properties of the algebra of all linear operators acting on the algebra of formal power series also from the standpoint of the Graves-Heisenberg-Weyl algebra.

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**Gabor Lippner**

**Multiple points of fold maps**

I would like to prove a generalisation of the Herbert multiple-point formula for fold maps. The problem is that the multiple-point manifolds of singular maps are generally not closed. This can be overcome by considering them relative to the singular set.

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**Sergey Melikhov**

**On link maps and embeddings in the 3/4 range**

In 2003, Goodwillie, Klein and Weiss obtained a classification of smooth embeddings in the 3/4 range in terms that involve limits of diagrams indexed by all points of a manifold. In this talk I hope to report on latest progress towards classifying PL and smooth embeddings in this range in terms of equivariant, stratum-preserving homotopy classes of the induced maps between appropriately compactified triple point configuration spaces. Recent results in this and related directions include:

- 1) The set of isotopy classes of embeddings of a polyhedron in  $R^m$  in the 2/3 (metastable) range, if nonempty, is in a bijection with a certain generalized cohomology group.
- 2) There is a “quadratic” compactification of the triple point configuration space of a smooth manifold (as opposed to the “linear” compactification of Fulton-MacPherson) that suffices to classify classical knots up to Goussarov-Habiro  $C_3$ -equivalence.
- 3) The triple  $\mu$ -invariant and the  $\beta$ -invariant of link maps can be described in terms of cohomological invariants of maps between suitable triple point configuration spaces.

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**Vladimir Nezhinskij**

**A generalization of Goussarov’s Groups for Higher-Dimensional Knots**

Our aim is to generalize (as far as it get possible) the Goussarov’s theory of  $n$ -equivalence of one-dimensional knots to knots of higher dimensions.

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**Arkadiy Skopenkov**

**Classification of smooth embeddings of 4-manifolds in the 7-space  
(joint work with M. Kreck)**

We work in the smooth category. Let  $N$  be a closed connected  $n$ -manifold and assume that  $m > n^2$ . Denote by  $Emb^m(N)$  the set of embeddings  $N \rightarrow R^m$  up to isotopy. The group  $Emb^m(S^n)$  acts on  $Emb^m(N)$  by embedded connected sum of a manifold and a sphere. If  $Emb^m(S^n)$  is non-zero (which often happens for  $2m < 3n^4$ ) then no results on this action and no complete description of  $Emb^m(N)$  were known. Our main results are *examples of the triviality and the effectiveness of this action*, and *a complete isotopy classification of embeddings into  $R^7$  for certain 4-manifolds  $N$* . The proofs are based on the first author’s modification of surgery theory and on construction of a new embedding invariant.

*Corollary.* For each embedding  $f : CP^2 \rightarrow R^7$  and each non-trivial knot  $g : S^4 \rightarrow R^7$  the embedding  $f \# g$  is isotopic to  $f$ .

## Mikhail Skopenkov

### A formula for the group of links in the 2-metastable dimension

We present a short proof of an explicit formula for the group of links (and also link maps) in the 2-metastable dimension. This improves a result of Haefliger from 1966.

*Theorem.* Assume that  $p \leq q \leq m - 3$  and  $2p2q \leq 3m - 7$ . Denote by  $L_{p,q}^m$  (resp.  $K_p^m$ ) the group of smooth embeddings  $S^p \sqcup S^q \rightarrow S^m$  (resp.  $S^p \rightarrow S^m$ ) up to smooth isotopy. Then

$$L_{p,q}^m \cong \pi_p(S^{m-q-1}) \oplus \pi_{pq2-m}(SO/SO_{m-p-1}) \oplus K_p^m \oplus K_q^m.$$

Our approach is based on an exact sequence involving the group of links, link maps and their relative versions. The latter are classified via the  $\beta$ -invariant of Koschorke, Habegger and Kaiser.

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## John Terilla

### A Smoothness Theorem for differential BV algebras

I will state and prove a new formality theorem for differential BV algebras which is relevant in algebraic quantum field theories arising from BV quantization. The theorem gives a simple necessary and sufficient condition for the existence of a versal action functional, from which one can construct a weak-Frobenius manifold structure. This necessary and sufficient condition is easily stated in terms of the existence of a certain chain map which notably makes no explicit reference to the Lie bracket.

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## Svetlana Terzic

### On computation of Chern numbers on some homogeneous spaces

In this talk we consider generalized symmetric spaces, i. e. homogeneous spaces for which the stabilizer appears as the fixed point subgroup of some finite order automorphism of the group. We show that, for these spaces, classical Cartan theorem together with Borel-Hirzebruch work on homogeneous spaces, yield to complete description of their invariant complex structures as well as computation of corresponding Chern classes. This approach makes also possible to compute explicitly their Chern numbers and, consequently, to discuss topological invariance for some of them.

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**Tamás Terpai**

**On the cobordism group of fold maps**

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**Ping Zhang**

**On Smooth Maps with Critical Points of a Manifold onto a Manifold  
(joint work with C. Pinteá)**

Given closed smooth manifolds  $M$  of dimension  $n$  plus  $k$  and  $N$  of dimension  $n$ . Among all smooth maps of  $M$  onto  $N$ , is there one with finitely many critical points? If the answer is yes, what is this number, or how is it estimated? If the answer is no, how is the size of the critical set measured? In this joint work with C. Pinteá we shall attempt to answer these questions with certain restrictions on the dimensions and algebraic invariants, exploiting classical results of Church and Timourian and improving recent results of Andrica and Funar.

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