RATIONALITY AND EXPONENTIAL GROWTH PROPERTIES OF THE BOUNDARY OPERATORS IN THE NOVIKOV COMPLEX.

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§1. Introduction

The classical Morse-Thom-Smale construction associates to a Morse function $g : M \to \mathbf{R}$ on a closed manifold a chain complex $C_*(g)$ of free abelian groups, where the number of free generators of $C_p(g)$ equals the number of critical points of g of index p for each p. The homology of this complex is isomorphic to the homology of the manifold , and the boundary operator in this complex is defined in a geometric way, using the algebraic number of trajectories of a gradient of g, joining critical points of g (see [5], [9], [13], [14], [15]).

In the early 80s S.P.Novikov generalized this construction to the case of maps $f : M \to S^1$ (see [6]). Here M is a closed connected manifold , $f : M \to S^1$ is a Morse map, non-homotopic to zero. The corresponding analog of Morse complex is a free chain complex $C_*(f)$ over the ring $\mathbf{Z}((t))$ of the formal power series with integer coefficients and finite negative part (that is $\mathbf{Z}((t)) = \{\sum a_n t^n \mid a_n \in \mathbf{Z} \text{ and } \exists N : a_n = 0 \text{ for } n < N\}$). The number of free generators of $C_p(f)$ equals the number of critical points of f of index p, and the homology of $C_*(f)$ equals to the completed homology of the corresponding cyclic covering of M.

The boundary operator in this complex depends on the choice of a riemannian metric on M or of a gradient-like vector field v for f. We prefer in our work the language of gradient-like vector fields (see §2 for precise definitions). To explicit the boundary operators, let $\bar{M} \xrightarrow{\mathcal{P}} M$ be the connected infinite cyclic covering for which $f \circ \mathcal{P}$ is homotopic to zero. Choose a lifting $F : \bar{M} \to \mathbf{R}$ of $f \circ \mathcal{P}$ and let t be the generator of the structure group $(\approx \mathbf{Z})$ of $\mathcal{P}$ such that for every $x \in \bar{M}$ we have $F(xt) < F(x)$. For every critical point x of f choose a lifiting $\bar{x}$ of x to $\bar{M}$. Choose orientations of stable manifolds of critical points. Then for every critical points x, y of f with $\operatorname{ind} x = \operatorname{ind} y + 1$, and every $k \in \mathbf{Z}$ the incidence coefficient $n_k(x, y; v)$ is defined as the algebraic number of $(-v)$-trajectories joining $\bar{x}$ to $\bar{y}t^k$, each trajectory being counted with the sign, arising from the choice of orientations. (Two trajectories which differ by a choice of parameter, are identified; v is supposed to satisfy the transversality assumption.) Set $n(x, y; v) = \sum_{k \in \mathbf{Z}} n_k(x, y; v) t^k \in \mathbf{Z}((t))$. Now the boundary operator $\partial : C_m(f) \to C_{m-1}(f)$ is defined by $\partial x = \sum_y y \cdot n(x, y; v)$, where x is a critical point of f of index m, and the sum ranges over critical points of f of index $m - 1$.

Since the beginning S.P.Novikov conjectured that the power series $n(x, y; v)$ had some nice analytic properties. In particular he conjectured that

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($\mathcal{E}$) *Generically the coefficients* $n_k(x, y; v)$ *grow at most exponentially with* k.

This conjecture is stated in [7] for the case of analytic Morse maps f. The present paper announces that generically the coefficients $n(x, y; v)$ are rational functions in t, which implies the exponential estimate (see §2 for precise statement).

Further, denote by $N_k(x, y; v)$ the *total* number of $(-v)$-trajectories joining $\bar{x}$ to $\bar{y}t^k$. The version of the conjecture ($\mathcal{E}$) presented in [1, p. 83] states that generically the numbers $N_k(x, y; v)$ grow at most exponentially with k. (There is also the corresponding statement for Morse forms.) Closely related to it is the question of V.I.Arnold about the asymptotic behaviour of $A^n X \cap Y$ where A is a diffeomorphism of a manifold M to itself, X, Y are submanifolds of M and $n \to \infty$ (see [1,2]).

We confirm the exponential estimate for $N_k(x, y; v)$ as well; see §4 for the precise statement as well as for the statement of the corresponding result for Morse forms.

Using the universal covering $\widetilde{M} \to M$ instead of the cyclic covering above, one obtains a version of Novikov complex defined over corresponding completion of the group ring $\mathbf{Z}\pi_1 M$. We announce in §3 that the corresponding incidence coefficients belong to the (non commutative) localization in the sense of P.M.Cohn of the ring $\mathbf{Z}\pi_1 M$.

Instead of Morse maps $M \to S^1$ one can consider Morse 1-forms (recall that Morse form is a closed 1-form, which is locally a differential of a Morse function). To each Morse form ω one associates the homomorphism of integration $H(\omega) : H_1(M, \mathbf{Z}) \to \mathbf{R}$ and the free $\mathbf{Z}^k$-covering $M' \to M$ corresponding to the kernel of the composition $\pi_1 M \to H_1(M, \mathbf{Z}) \xrightarrow{H(\omega)} \mathbf{R}$ (where $k = \text{rk Im } H(\omega)$). The corresponding incidence coefficients belong to a suitable completion of the Laurent polynomial ring $\mathbf{Z}[\mathbf{Z}^k]$. We announce that generically they belong to the corresponding localization of $\mathbf{Z}[\mathbf{Z}^k]$ (see Theorem 3 of the present paper, which states a bit stronger assertion, concerning the maximal free abelian coverings).

The full proof of Theorem 1 is contained in [11]. The full proofs of the results of §3,4 are contained in [12].

§2. Incidence coefficients with values in $\mathbf{Z}((t))$

We need some definitions. Let $f : M \to \mathbf{R}^1$ be a Morse function on a manifold M, $\dim M = n$. The set of critical points of f will be denoted by $S(f)$. Let v be a vector field on M (we assume that all objects are of class C^∞). We say that v is an f*-gradient* if for every non-critical point x of f we have $df(v)(x) > 0$ and for every critical point p of f there is a chart $\Phi : W \to V$, where W is an open neighborhood of p, and V is an open neighborhood of 0 in $\mathbf{R}^n$, such that $\Phi(p) = 0$ and

(1) $(f \circ \Phi^{-1})(x_1, ..., x_n) = f(p) + \sum_{i=1}^{n} \alpha_i x_i^2$ with $\alpha_m < 0$ for $m \leqslant k$ and $\alpha_m > 0$ for $m > k$.

(2) $\Phi_*(v) = (-x_1, ..., -x_k, x_{k+1}, ..., x_n)$.

(here k is the index of the critical point p). The set of all f-gradients will be denoted by $\mathcal{G}(f)$, and the set of all f-gradients satisfying the transversality assumption will be denoted by $\mathcal{G}t(f)$. We assume similar terminology for maps $f : M \to S^1$.

Theorem 1. *Let M be a closed connected manifold , $f : M \to S^1$ be a Morse map, non homotopic to zero. Then in the set $\mathcal{G}t(f)$ there is a subset $\mathcal{G}t_0(f)$ with the following properties:*

1. *$\mathcal{G}t_0(f)$ is open and dense in $\mathcal{G}t(f)$ with respect to C^0 topology.*
2. *If $v \in \mathcal{G}t_0(f)$, $x, y \in S(f)$ and $\operatorname{ind} x = \operatorname{ind} y + 1$, then $\sum_{k\in\mathbf{Z}} n_k(x, y; v)t^k$ is a rational function of t of the form $\frac{P(t)}{t^m Q(t)}$, where $P(t)$ and $Q(t)$ are polynomials with integral coefficients, $m \in \mathbf{N}$, and $Q(0) = 1$.*
3. *Let $v \in \mathcal{G}t_0(f)$. Let U be a neighborhood of $S(f)$. Then for every $w \in \mathcal{G}t_0(f)$ such that $w = v$ in U and w is sufficiently close to v in C^0 topology we have: $n_k(x, y; v) = n_k(x, y; w)$ for every $x, y \in S(f)$ with $\operatorname{ind} x = \operatorname{ind} y + 1$, and every $k \in \mathbf{Z}$.*

Remarks. 1) The exponential estimate in $(\mathcal{E})$ follow immediately, since the the Taylor series of every rational function of the form $\frac{P(t)}{Q(t)}$ with $Q(t) \neq 0$ has a non-zero radius of convergency.

2) From the main theorem of [10] it follows that for every finite sequence $a_1, ..., a_k$ of integers there is a Morse map $f : M \to S^1$ on a manifold M, an f-gradient v and critical points x, y of f with $\operatorname{ind} x = \operatorname{ind} y + 1$ such that $n_0(x, y; v) = 1$ and $n_i(x, y; v) = a_i$ for $1 \leqslant i \leqslant k$.

§3. Incidence coefficients with values in completions of group rings

To state the results we recall some algebraic and Morse-theoretic definitions.

Let G be a group and $\xi : G \to \mathbf{R}$ be a group homomorphism . We denote by $(\mathbf{Z}G)\hat{}\hat{}$ the abelian group of all formal linear combinations $\sum\limits_{g\in G} n_g g$, infinite in general (where $n_g \in \mathbf{Z}$). Novikov ring $\mathbf{Z}G^-_\xi$ is the ring of such $\lambda \in (\mathbf{Z}G)\hat{}\hat{}$, that for every $c \in \mathbf{R}$ the set $\operatorname{supp} \lambda \cap \xi^{-1}([c, \infty[)$ is finite.

Let ω be a closed 1-form on a manifold M. The deRham cohomology class of ω will be denoted by $[\omega]$ and the corresponding homomorphism $\pi_1 M \to \mathbf{R}$ will be denoted by $\{\omega\}$. We say that ω is a *Morse form* , if locally it is the differential of a Morse function. The set of zeros of ω will be denoted by $S(\omega)$. A Morse form ω is the differential of a Morse map $f : M \to S^1$ if and only if $[\omega] \in \operatorname{Im} (H^1(M, \mathbf{Z}) \to H^1(M, \mathbf{R}))$.

An obvious generalization of the definition of f-gradient for Morse maps to S^1 gives the notion of *ω-gradient* of a Morse form ω. The set of all ω-gradients will be denoted by $\mathcal{G}(\omega)$ and the set of all ω-gradients satisfying the transversality assumption will be denoted by $\mathcal{G}t(\omega)$. Let ω be a Morse form on a closed connected manifold M and let $v \in \mathcal{G}t(\omega)$. For each zero x of ω choose a lifting $\widetilde{x}$ of x to $\widetilde{M}$ and an orientation of the stable manifold of x. Then for every $x, y \in S(\omega)$ with $\operatorname{ind} x = \operatorname{ind} y + 1$ the incidence coefficient $\widetilde{n}(\widetilde{x}, \widetilde{y}; v) \in (\mathbf{Z}\pi_1 M)^-_\xi$ can be defined (the definition is similar to that of $n(x, y; v)$ in §1; for the case of Morse maps $M \to S^1$ one can find it in [9].)

(Although we shall not use the notion of Novikov complex in this paper, we give the definition. Let $C_p(\omega)$ be the free right $\mathbf{Z}(\pi_1 M)^-_\xi$-module, generated by $S_p(\omega)$, where $S_p(\omega)$ stands for the set of zeros of ω of index p. Define a homomorphism $\partial_p : C_p(\omega) \to C_{p-1}(\omega)$

by $\partial_p x = \sum\limits_{y\in S_{p-1}(\omega)} y\cdot \widetilde{n}(\widetilde{x},\widetilde{y};v)$, where $x\in S_p(\omega)$. One can prove that $\partial_p\circ\partial_{p+1}=0$; the resulting complex is called *Novikov complex* .)

1. Morse maps $M\to S^1$

Let $\xi: G\to \mathbf{Z}$ be a group epimorphism. Denote Ker ξ by H. For $n\in\mathbf{Z}$ denote $\xi^{-1}(n)$ by $G_{(n)}$ and $\{x\in \mathbf{Z}G \mid \operatorname{supp} x\subset G_{(n)}\}$ by $\mathbf{Z}G_{(n)}$. Denote $\xi^{-1}(]-\infty,-1])$ by G_- and $\{x\in\mathbf{Z}G\mid \operatorname{supp} x\subset G_-\}$ by $\mathbf{Z}G_-$. Choose $\theta\in\mathbf{Z}G_{(-1)}$. It is easy to see that $(\mathbf{Z}G)^-_\xi$ is identified with the ring of power series of the form $\sum\limits_{i=-\infty}^{\infty} a_i\theta^i$, where $a_i\in\mathbf{Z}H$ and the negative part of the series is finite.

Set $\Sigma_n=\{\mathbf{1}+A\mid A\in Mat_{n\times n}(\mathbf{Z}G_{(-1)})\}$. Set $\Sigma=\bigcup\limits_{n\geqslant 1}\Sigma_n$.

There is the corresponding localization ring $\mathbf{Z}G_\Sigma$ (see [4, p.255]). Every matrix in Σ_n is invertible in $Mat_{n\times n}(\mathbf{Z}G^-_\xi)$, the inverse of $\mathbf{1}+A$ being given by $\sum_{n=0}^\infty(-1)^nA^n$, therefore the localization map $\lambda:\mathbf{Z}G\to\mathbf{Z}G_\Sigma$ is injective and the inclusion $i:\mathbf{Z}G\hookrightarrow\mathbf{Z}G^-_\xi$ factors through a ring homomorphism $\ell:\mathbf{Z}G_\Sigma\to\mathbf{Z}G^-_\xi$.

Proceeding to the statement of the next result, let M be a connected closed manifold and $f:M\to S^1$ be a Morse map, nonhomotopic to zero. Denote by ξ the induced homomorphism $\pi_1M\to\mathbf{Z}$. Assume that ξ is epimorphic. Then by the previous discussion we have the localization $(\mathbf{Z}\pi_1M)_\Sigma$ and the homomorphism $\ell:(\mathbf{Z}\pi_1M)_\Sigma\to(\mathbf{Z}\pi_1M)^-_\xi$.

Theorem 2. *In $\mathcal{G}t(f)$ there is a subset $\mathcal{G}t_1(f)$ with the following properties:*

1. *$\mathcal{G}t_1(f)$ is open and dense in $\mathcal{G}t(f)$ with respect to C^0 topology.*
2. *If $v\in\mathcal{G}t_1(f)$ then for every $x,y\in S(f)$ with $\operatorname{ind}x=\operatorname{ind}y+1$ we have $\widetilde{n}(\widetilde{x},\widetilde{y};v)\in$ Im ℓ.*
3. *Let $v\in\mathcal{G}t_1(f)$. Let U be a neighborhood of $S(f)$. Then for every $w\in\mathcal{G}t_1(f)$ such that $w=v$ in U and w is sufficiently close to v in C^0 topology we have: $\widetilde{n}(\widetilde{x},\widetilde{y};v)=\widetilde{n}(\widetilde{x},\widetilde{y};w)$ for every $x,y\in S(f)$ with $\operatorname{ind}x=\operatorname{ind}y+1$.*

2. Morse forms within arbitrary cohomology classes

Let ω be a Morse form on a closed connected manifold M and let $\phi:\overline{\overline{M}}\to M$ be any connected regular covering with structure group G such that $\phi^*([\omega])=0$. Then the homomorphism $\{\omega\}:\pi_1M\to\mathbf{R}$ factors as $\pi_1M\to G\xrightarrow{\langle\omega\rangle}\mathbf{R}$. Let $v\in\mathcal{G}t(\omega)$. For every $x\in S(\omega)$ choose a lifting $\overline{\overline{x}}$ of x to $\overline{\overline{M}}$ and an orientation of the stable manifold of x. Then for every $x,y\in S(\omega)$ with $\operatorname{ind}x=\operatorname{ind}y+1$ the incidence coefficient $\overline{\overline{n}}(\overline{\overline{x}},\overline{\overline{y}};v)\in\mathbf{Z}G^-_{\langle\omega\rangle}$ is defined (similarly to $\widetilde{n}(\widetilde{x},\widetilde{y};v)$).

In particular it is the case for the maximal free abelian covering $\widehat{M}\xrightarrow{\mathcal{P}}M$ with the structure group $H_1(M,\mathbf{Z})/\text{Tors}\approx\mathbf{Z}^m$. The corresponding homomorphism $\mathbf{Z}^m\to\mathbf{R}$ is the one arising from the de Rham cohomology class $[\omega]$; it will be denoted by the same symbol $[\omega]$. Set $S_{[\omega]} = \{P \in \mathbf{Z}[\mathbf{Z}^m] \mid P = \mathbf{1} + Q \text{ and } \operatorname{supp} Q \subset [\omega]^{-1}(]-\infty, 0[)\}$.

Theorem 3. *Let ω be a Morse form with $[\omega] \neq 0$. Then there is a subset $\mathcal{G}t_1(\omega) \subset \mathcal{G}t(\omega)$ with the following properties:*

(1) *$\mathcal{G}t_1(\omega)$ is open and dense in $\mathcal{G}t(\omega)$ with respect to C^0 topology.*
(2) *For every $v \in \mathcal{G}t_1(\omega)$ and every $x, y \in S(\omega)$ with $\operatorname{ind} x = \operatorname{ind} y + 1$ we have: $\widehat{n}(\widehat{x}, \widehat{y}; v) \in S_{[\omega]}^{-1}\mathbf{Z}[\mathbf{Z}^m]$.*
(3) *Let $v \in \mathcal{G}t_1(\omega)$. Let U be a neighborhood of $S(\omega)$. Then for every $w \in \mathcal{G}t_1(\omega)$ such that $w = v$ in U and w is sufficiently close to v in C^0 topology we have: $\widehat{n}(\widehat{x}, \widehat{y}; v) = \widehat{n}(\widehat{x}, \widehat{y}; w)$ for every $x, y \in S(\omega)$ with $\operatorname{ind} x = \operatorname{ind} y + 1$.*

3. Exponential growth estimates

Let G be a group. For an element $a = \sum n_g g \in \mathbf{Z}G$ we denote by $\|a\|$ the sum $\sum |n_g|$.

Let $\xi : G \to \mathbf{R}$ be a homomorphism . For $\lambda = \sum_g n_g g \in \mathbf{Z}G_\xi^-$ and $c \in \mathbf{R}$ we denote by $\lambda[c]$ the element $\sum_{\xi(g) \geqslant c} n_g g$ of $\mathbf{Z}[\pi_1 M]$ and we set $N_c(\lambda) = \|\lambda[c]\|$. We shall say that λ is of exponential growth if there are $A, B \geqslant 0$ such that for every $c \in \mathbf{R}$ we have $N_c(\lambda) \leqslant Ae^{-cB}$. It is easy to prove that the elements of exponential growth form a subring of $\mathbf{Z}G_\xi^-$, which contains $\mathbf{Z}G$.

Theorem 4. *Let ω be a Morse form with $[\omega] \neq 0$, and let v be an ω-gradient, belonging to $\mathcal{G}t_1(\omega)$. Let $x, y \in S(\omega), \operatorname{ind} x = \operatorname{ind} y + 1$. Then $\widetilde{n}(\widetilde{x}, \widetilde{y}; v)$ is of exponential growth .*

§4 Exponential growth estimate of the total number of trajectories

1. Morse maps $M \to S^1$

We assume here the terminology of §1. Recall from [10,§2B] that an f-gradient v is called *good* if for every $p, q \in S(f)$ we have

$$\Big(\operatorname{ind} p \leqslant \operatorname{ind} q + 1\Big) \Rightarrow \Big(D(p, v) \pitchfork D(q, -v)\Big).$$

The set of all good f-gradients will be denoted by $\mathcal{G}d(f)$. For $p, q \in S(F)$ with $\operatorname{ind} p = \operatorname{ind} q + 1$ and for $v \in \mathcal{G}d(f)$ the set of $(-v)$- trajectories joining p to q is finite. For $x, y \in S(f)$ with $\operatorname{ind} x = \operatorname{ind} y + 1$ and $k \in \mathbf{Z}$ denote by $N_k(x, y; v)$ the number of $(-v)$-trajectories joining $\bar{x}$ to $\bar{y}t^k$ (recall that $\bar{x}, \bar{y}$ stand for the chosen liftings of x, y to $\bar{M}$).

Theorem 5. *In the set $\mathcal{G}(f)$ there is a subset $\mathcal{G}_0(f)$ with the following properties:*

(1) *$\mathcal{G}_0(f)$ is C^0 dense in $\mathcal{G}(f)$ and $\mathcal{G}_0(f) \subset \mathcal{G}d(f)$.*
(2) *Let $v \in \mathcal{G}_0(f)$. Then there are constants $C, D > 0$ such that for every $x, y \in S(f)$ with $\operatorname{ind} x = \operatorname{ind} y + 1$ and for every $k \in \mathbf{Z}$ we have $N_k(x, y; v) \leqslant C \cdot D^k$.*

6. Morse forms within arbitrary cohomology classes

We assume here the terminology of §3. Further, for a Morse form ω on M we denote by $\mathcal{G}d(\omega)$ the set of all the good ω-gradients . (The definition of a good ω-gradient is similar to that of good f-gradient from the above.)

For any $v \in \mathcal{G}d(\omega)$, any $x, y \in S(\omega)$ with $\mathrm{ind}x = \mathrm{ind}y + 1$ and any $g \in \pi_1 M$ the set of $(-v)$- trajectories joining $\widetilde{x}$ to $\widetilde{y}g$ is finite and we denote its cardinality by $N(\widetilde{x}, \widetilde{y}, g; v)$. For $c \in \mathbf{R}$ we denote by $N_{\geqslant c}(\widetilde{x}, \widetilde{y}; v)$ the sum $\sum_{g:\{\omega\}(g)\geqslant c} N(\widetilde{x}, \widetilde{y}, g; v)$.

Theorem 6. *In the set $\mathcal{G}(\omega)$ there is a subset $\mathcal{G}_0(\omega)$ with the following properties:*

(1) *$\mathcal{G}_0(\omega)$ is dense in $\mathcal{G}(\omega)$ with respect to C^0 topology; $\mathcal{G}_0(\omega) \subset \mathcal{G}d(\omega)$.*
(2) *Let $v \in \mathcal{G}_0(\omega)$. There are constants $C, D > 0$ such that for every $x, y \in S(\omega)$ with $\mathrm{ind}x = \mathrm{ind}y + 1$ and every $\lambda \in \mathbf{R}$ we have $N_{\geqslant\lambda}(x, y; v) \leqslant C \cdot D^{-\lambda}$.*

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