Programme de la réunion annuelle du GDR 2022 à Nantes

October 20, 2022

1 Minicourse

1.1 Constanze Roitzheim (University of Kent)

Title: Introduction to Chromatic Homotopy Theory

The goal of the talks is to introduce some of the foundations of chromatic stable homotopy theory and to explain how these methods are used to study the structure of the stable homotopy category. If time permits, we will look at some more recent results at the end.

Topics will include:

- left Bousfield localisation,
- p-localisation, p-completion, rationalisation, K-theory,
- $E(n)$ and $K(n)$: construction and properties,
- nilpotence, periodicity, chromatic convergence,
- thick subcategories and the Balmer spectrum.

2 Invited Lectures

2.1 Vladimir Dotsenko (Université de Strasbourg)

Title: Subalgebras of free algebras

Abstract: The celebrated theorem of Shirshov and Witt from 1950s asserts that every subalgebra of every free Lie algebra is itself free. Since then, ring theorists have been searching for other types of algebras satisfying this property, and such types of algebras have been perceived as extremely rare. I shall explain a new approach to this property using operads and homotopy theory which gives some

concrete criteria allowing one to check this property, and to exhibit infinitely many new types of algebras for which it holds. This is based on a joint paper with Ualbai Umirbaev.

2.2 Yonatan Harpaz (Université Paris 13)

Title: The infinitesimal tangle hypothesis

Abstract: The tangle hypothesis is a variant of the cobordism hypothesis pertaining to tangles, that is, manifolds and cobordisms embedded in Euclidean spaces (and equipped with a suitable type of framing), in a manner that the codimension of the embedding remains fixed. This elaborate ensemble of geometric data can be assembled into a single higher categorical object, an E_m-monoidal (∞, n)-category $\mathrm{Tang}_{n,m}$, where n is the maximal dimension of cobordisms one is considering and m is the fixed codimension of all the Euclidean embeddings. The tangle hypothesis then asserts that $\mathrm{Tang}_{n,m}$ is freely generated as an E_m-monoidal (∞, n)-category with duals from a single object. Equivalently, $B^m\mathrm{Tang}_{n,m}$ is feely generated as an $(\infty, n+m)$-category with adjoints from an object equipped with an m-cell.

A sketch of proof of the tangle hypothesis appears in Lurie's 2009 text, and a conditional argument reducing the tangle hypothesis to another conjecture was later provided by Ayala and Francis, though a complete and formal proof has not yet appeared. In this talk I will describe work in progress with Joost Nuiten which provides an infinitesimal version of this conjecture. More precisely, applying previous work with Joost Nuiten and Matan Prasma on Quillen cohomology of higher categories we calculate the cotangent complex of $B^m\mathrm{Tang}_{n,m}$ and show that, in a suitable sense, it is freely generated from a single m-cell. This can be considered as supporting evidence in the direction of the tangle hypothesis, but also reduces the tangle hypothesis to a statement on the level of $(m+n+1, m+n)$-categories using a form of obstruction theory for higher categories.

2.3 Danica Kosanovic (ETH Zürich)

Title: Homotopy groups of some embedding spaces

Abstract: In this talk I will present joint work with Peter Teichner, in which we give a reinterpretation and extension of the work of Jean-Pierre Dax from the 1970s, on the lowest interesting homotopy group of spaces of embeddings of a disk into a manifold. We combine this with a "delooping" result (space level light bulb theorem), to obtain in certain cases interesting computations beyond Dax's metastable range.

2.4 Johan Leray (Nantes Université)

Title: Les algèbres pré-Calabi–Yau sont les gèbres double-Poisson à homotopie près.

Abstract: En géométrie non-commutative dérivée, deux notions de structure de Poisson non–commutative homotopique coexistent : d'une part, les structures pré-Calabi–Yau, définies comme des éléments de Maurer–Cartan d'une généralisation des champs de polyvecteurs non commutatifs ; d'autre part, les structures double-Poisson infinies, définies comme la version à homotopie près des double-crochets de Poisson de Van den Bergh. Dans un récent travail en collaboration avec B. Vallette, nous démontrons qu'il y a correspondance de ces deux types de structures, généralisant ainsi des résultats de Iyudu–Kontsevich–Vlassopoulos et Fernandez–Herscovich. Cette correspondance nous permet notamment de définir une bonne notion de morphismes entre algèbres pré-Calabi–Yau. Après avoir introduit les deux notions, je présenterai la stratégie de preuve de cette correspondance, ainsi que les conséquences de celle-ci.

2.5 Lyne Moser (Universität Regensburg)

Title: Defining limits in (∞, n)-categories

Abstract: Limits in an $(\infty, 1)$-category have been constructed as terminal objects in the corresponding $(\infty, 1)$-category of cones. In this talk, I will present a generalization of this construction to the (∞, n)-categorical setting for higher n, focusing on the case where $n = 2$. This is work in progress with Nima Rasekh and Martina Rovelli.

Unlike their 1-categorical analogues, limits in a (strict) 2-category cannot be characterized as terminal objects in the corresponding 2-category of cones. Instead, a passage to double categories allows for such a characterization. Inspired by the strict case, we define $(\infty, 2)$-limits as terminal objects in a double $(\infty, 1)$-category of cones and show that this definition is equivalent to the established definition of $(\infty, 2)$-limits as $(\infty, 1)$-categorically enriched limits.

2.6 Neil Strickland (University of Sheffield)

Title: Chromatic cohomology of finite general linear groups

Abstract: There has been great interest in studying the Morava E-theory of classifying spaces of finite groups. There are a number of interesting theorems that apply to all finite groups, and also calculations for specific cases, including abelian groups, symmetric groups and various groups of small nilpotence class. In this talk we will discuss the case of general linear groups over finite fields of characteristic different to that of the Morava E-theory. Here there is a description of the answer due to Tanabe, but it is not very explicit or tractable.

We will use different methods which provide an explicit basis and make contact with the symmetric group case via the geometric theory of formal schemes. There are several lines of evidence that point the way to the main theorem, but the only one that we could convert into an actual proof involves an elaborate calculation with the Atiyah-Hirzebruch spectral sequence. This is joint work with Sam Marsh and Sam Hutchinson.

3 Proposed lectures

3.1 Gabriel Angelini-Knoll (Université Paris 13)

Title: Algebraic K-theory of elliptic cohomology

Abstract: The algebraic K-theory of finite fields, the integers, and complex K-theory are some of the most fundamental algebraic K-theory computations in the field. I will discuss an explicit computation of higher chromatic complexity, the algebraic K-theory of the second truncated Brown–Peterson spectrum. As a consequence, we prove a strong form of the red-shift conjecture for the second truncated Brown–Peterson spectrum, refining a recent result of Hahn-Wilson in this case. This is based on joint work with C. Ausoni, D. Culver, E. Höning, and J. Rognes.

3.2 El Mehdi Cherradi (Université Paris Cité)

Title: The smothering model structure and indiscernible isomorphisms

Abstract: The notion of smothering functor introduced by Emily Riehl and Dominic Verity defines a class of functors which extends the usual notion of equivalence of categories by relaxing the faithfulness condition. These functors appear naturally as comparison functors between homotopy categories, for instance the comparison functor relating the homotopy category of a pullback of quasicategories to the pullback of the homotopy categories of these quasicategories. In this work, we construct a right Bousfield localization of the natural model structure on Cat, whose weak equivalences are a slight strengthening of the original definition of smothering functors. Interestingly, it is possible to characterize those weak equivalences by relying on an equivalence relation between isomorphisms of a given category, called indiscernibility. This notion of indiscernible isomorphism sheds light on the strength of the homotopy relation between morphisms in a model category or more generally a quasicategory.

3.3 Basile Coron (Université de Strasbourg)

Title: Structures opéradiques sur les matroïdes et dualité de Koszul

Abstract: Il est connu que les inclusions de strates de compactifications merveilleuses de certaines familles d'arrangements d'hyperplans donnent des structures

opéradiques (opérade symétrique HyperCom, permutades de Losev Manin, brick manifolds etc). Le premier objectif de cet exposé est de montrer comment unifier ces exemples en une seule structure à l'aide du formalisme des catégories de Feynman et cela à un niveau purement combinatoire. Une fois ce but atteint on montrera comment la relation de dualité de Koszul HyperCom/Grav peut être étendue à ce cadre plus vaste.

3.4 Lucas Darbas (Nantes Université)

Title: Higher order Hochschild cohomology of schemes

Abstract: We define in two ways higher order Hochschild cohomology of schemes over a field of characteristic zero, first introduced by Pirashvili for commutative algebras [Pir00]. First, we can take the hyperext functor of the sheaf associated to the higher order Hochschild presheaf. In the separated smooth case, we obtain a Hodge decomposition which generalizes Pirashvili's Hodge decomposition. We can also define the Higher Hochschild cohomology of order d by taking the ext functor of the structure presheaf over the Higher Hochschild presheaf of order $d-1$. These two definitions are really close to those of Swan [Swa96] for classical Hochschild cohomology but tools are model categories and derived functors.

[Pir00] T. Pirashvili. Hodge decomposition for higher order Hochschild homology. Ann. Sci. Ec. Norm. Supér. (4), 33(2):151–179, 2000.

[Swa96] R. G. Swan. Hochschild cohomology of quasiprojective schemes. J. Pure Appl. Algebra, 110(1):57–80, 1996.

3.5 Jacques Darné (Université Catholique de Louvain)

Title: Lower central series of partitioned braids on surfaces

Abstract : Partitioned braid groups (sometimes called "mixed braid groups") are subgroups of the braid group standing between the pure braid group P_n and the whole braid group B_n. On the one hand, the lower central series of B_n is almost trivial. On the other hand, the lower central series of P_n is a very rich object, encoding finite type invariants of braids. As a consequence, one can expect partitioned braid groups to display a range of intermediate behaviors, and this is indeed what we observe. In this talk, we will explore these different behaviours and give an answer to the first question one can ask about these lower central series : when do they stop ? Even this simple question turns out to be a difficult one, especially when one considers its generalization to braids on surfaces. However, we will be able to answer it almost completely, leaving open only some cases of partitioned braids on the projective plane.

3.6 Sophie d'Espalungue (Université de Lille)

Title: Recollements d'associahèdres et catégories lax-monoidales (itérées) non biaisées

Abstract : On construit une catégorie d'arbres Υ que l'on munie d'un foncteur $K : \Upsilon \to \mathrm{Top}$, tel que la réalisation géométrique induite par K sur les préfaisceaux $[\Upsilon^{op}, \mathrm{Sets}]$ donne précisément l'associahèdre d'arité n lorsqu'appliquée au préfaisceau représenté par la corolle d'arité n, donnant ainsi aux associahèdres le rôle de cellules de base. On munit la catégorie Υ d'une topologie, de telle sorte que la réalisation géométrique d'un faisceau fournisse un polytope dont les faces sont des produits de faces de dimensions inférieures.

On remarquera que la suite formée des sous catégories pleines $\Upsilon(n)$ de Υ dont les objets sont les arbres d'arité n a une structure d'opérade dans la catégorie des catégories. De cette structure on tirera également une structure d'opérade sur la catégorie PSh_Υ des préfaisceaux sur Υ, au moyen d'une généralisation du produit de convolution de Day à une 2-catégorie autre que celle des catégories (en l'occurence, à la 2-catégorie des suites de catégories), que l'on exposera si le temps le permet. On pourra alors introduire la notion d'opérade interne à l'opérade PSh_Υ, dont la suite $\{\Upsilon(n)(-, Y_n)\}_n$ des représentables en la corolle Y_n d'arité n est un exemple (que l'on appellera A_Υ). On s'intéressera alors aux algèbres sur cette opérade dans diverses catégories: on retrouvera la notion d'A_∞ - algèbre dans la catégorie des complexes de chaines et dans la catégorie des espaces topologiques. On s'attardera alors à ces algèbres dans la catégorie des catégories, réalisant en particulier la notion de catégorie lax monoidale non biaisée (de Tom Leinster, HOpHCat) comme algèbre dans Cat sur l'opérade A_Υ. Si le temps le permet, on montrera comment les notions de catégories lax-monoidales n-fois itérées non biaisées peuvent se déduire des considérations précédentes à partir d'un produit tensoriel de Boardman-Vogt relâché, que l'on définira pour les opérades à valeurs catégoriques. On établira également les raisons qui nous portent à croire que l'étude des structures non biaisées fournissent un cadre approprié à l'étude des espaces de lacets itérés.

3.7 Clémentine Lemarié–Rieusset (Université de Bourgogne)

Title: Motivic knot theory

Abstract: In this talk I will present a new application of motivic homotopy theory: motivic knot theory. This theory, which I am exploring during my PhD, aims at finding analogues of certain knot (and link) invariants in algebraic geometry. In my talk I will briefly recall some notions from knot theory and motivic homotopy theory, then I will present an analogue of the linking number of two oriented disjoint knots and its computation in two examples.

3.8 Nguyen Xuân Bách

Title: Generalized Clifford algebras

Abstract: The Clifford algebra of a quadratic form is a very classical object, which appears in many contexts, such as Bott's periodicity in algebraic topology or the classification of quadratic forms. Several generalizations of this construction have been studied, for example, by replacing quadratic forms by polynomial forms of higher degree. The study of these generalizations suggest that maybe they are defined as naturally attached to certain algebraic varieties. In this talk, we review the classical Clifford algebra and give the construction of generalized Clifford algebra in terms of adjunction in higher category theory.

3.9 Ismail Razack (Université de Picardie Jules Verne)

Title: Hochschild cohomology of intersection algebra

Abstract: The intersection algebra of a smooth manifold consists of the singular chains algebra (or singular cochains) equipped with the intersection product (or the cup product). The Poincaré duality implies the existence of algebraic structures (Batalin-Vikovisky) on the Hochschild cohomology of this algebra. A topological interpretation is given via loop spaces. In this talk, we are interested in the study of topological spaces with singularities. In general, for these spaces, the Poincaré duality is not satisfied. In order to restore it, M. Goresky and R. MacPherson introduced the intersection complexes which have the structure of a perverse differential graded algebra. We will define the Hochschild (co)homology for this type of objets and we will explain when we get a Batalin-Vilkovisky algebra. We will present these non trival structures through some examples.

3.10 Victor Saunier (Université Paris 13)

Title: Une généralisation du théorème fondamental de la K-théorie algébrique aux infinie-catégories stables et aux foncteurs Verdier-localisants.

Abstract: L'espace de K-théorie d'une infinie-catégorie stable est un foncteur qui jouit d'une propriété universelle, connue depuis des travaux de Blumberg-Gepner-Tabuada (2013). Cette propriété de localisation, pour laquelle la K-théorie est universelle, est la clé d'une formule connue sous le nom de formule de Bass-Heller-Swan ou théorème fondamental de la K-théorie algébrique dans le cadre des anneaux, qui est une des rares formules générales de la K-théorie. En généralisant une preuve connue dans le contexte du foncteur A de K-théorie algébrique des espaces, on peut obtenir cette formule dans le cadre des infinie-catégories stables pour non seulement la K-théorie algébrique, mais aussi l'homologie de Hochschild topologique (THH) et l'homologie cyclique topologique (TC) qui partagent cette propriété de localisation.