A numerical scheme for a kinetic model for mixtures in the diffusive limit using the moment method

Andrea Bondesan*, Laurent Boudin†, Bérénice Grec‡

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In this talk [1], we consider a multi-species kinetic model in a one-dimensional setting with a diffusive scaling. More precisely, assuming that both Knudsen and Mach numbers of order $\varepsilon > 0$, we can write, for each species $1 \leq i \leq p$, the Boltzmann equation for mixtures [3] on the distribution function $f_i^\varepsilon = f_i^\varepsilon(t, x, v)$

$$\varepsilon \partial_t f_i^\varepsilon + v \partial_x f_i^\varepsilon = \frac{1}{\varepsilon} \sum_{k=1}^{p} Q_{ik}(f_i^\varepsilon, f_k^\varepsilon), \qquad \text{in } \mathbb{R}_+ \times \Omega \times \mathbb{R},$$

where Q_{ik} is the multi-species Boltzmann collision operator, and Ω is an open interval on $\mathbb{R}$. For each species, macroscopic quantities, such as its concentration and its flux, can be obtained as the first moments of its distribution function

$$c_i^\varepsilon(t, x) = \int_{\mathbb{R}} f_i^\varepsilon(t, x, v) \mathrm{d}v, \qquad c_i^\varepsilon(t, x) u_i^\varepsilon(t, x) = \int_{\mathbb{R}} v \, f_i^\varepsilon(t, x, v) \mathrm{d}v.$$

Formally, it has been proved [2] that the moments of the solutions to the Boltzmann equation for mixtures converge to $(c_i, c_i u_i)$ solutions of the Maxwell-Stefan equations when ε tends to zero, in the equimolar diffusion setting. The Maxwell-Stefan equations are then written

$$\partial_t c_i + \partial_x (c_i u_i) = 0, \qquad \partial_x c_i = - \sum_{k \neq i} \frac{c_k c_i u_i - c_i c_k u_k}{D_{ik}},$$

where the coefficients $D_{ik} > 0$ are binary diffusion coefficients between species i and k. This formal convergence is obtained by the moment method [5], relying on an ansatz on the distribution functions that they are at local Maxwellian states with different macroscopic velocities $\varepsilon u_i^\varepsilon$ of order ε.
In the context of kinetic equations for mixtures, previous approaches [4] are not adapted to the diffusive scaling we are dealing with. We therefore propose a new numerical scheme, mimicking the analytical analysis (moment method), approximating the solutions of the kinetic model both in a rarefied regime and in the diffusion limit. We prove some *a priori* estimates (mass conservation and nonnegativity), as well as existence of a solution to the numerical scheme. We also present numerical simulations illustrating the asymptotic-preserving behavior of the scheme, and its capacity to describe uphill diffusion phenomena for mixtures.

*MAP5, Université Paris Descartes, F-75006 PARIS, andrea.bondesan@parisdescartes.fr
†LJLL, Sorbonne Université, F-75006 PARIS, laurent.boudin@sorbonne-universite.fr
‡MAP5, Université Paris Descartes, F-75006 PARIS, berenice.grec@parisdescartes.fr

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