ANALYSIS OF A FINITE-VOLUME SCHEME FOR A CROSS-DIFFUSION SYSTEM MODELING BIOFILM GROWTH

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Biofilms are organized cooperating communities of microorganisms. Typically, biofilms consist of several species such that multicomponent fluid models need to be considered. Recently, a cross-diffusion biofilm model was introduced by Rahman, Sudarsan, and Eberl [3]. It was derived formally from a space-time discrete random walk on a lattice. The model equations for the densities of the biofilm species u_i are given by

$$\partial_t u_i + \operatorname{div} \mathcal{F}_i = 0, \quad \mathcal{F}_i = -\alpha_i p(M)^2 \nabla \frac{u_i q(M)}{p(M)} \quad \text{in } \Omega,\ t > 0,\ i = 1, \ldots, n, \tag{1}$$

where $\Omega \subset \mathbb{R}^d$ $(d \geq 1)$ is a bounded domain, $\alpha_i > 0$ are some diffusion coefficients, p and q are some regular functions defined on $[0,1]$ with p decreasing such that $p(1) = 0$ and $M = \sum_{i=1}^n u_i$ is the total biomass. Equations (1) are supplemented with initial and mixed Dirichlet-Neumann boundary conditions. The global existence of nonnegative weak solutions and the long time behavior of solutions to (1) are established in [1]. The key of the analysis of this cross-diffusion system, is that system (1) admits a formal gradient-flow structure.

In this talk, I will introduce and analyze a finite volume scheme which preserves the structure of the continuous model (1). It consists in a Euler discretization in time and a two-point flux approximation in space. I will prove the existence of finite-volume solutions with nonnegative discrete densities $u_{i,K}$ and discrete total biomass $M_K < 1$ for all control volumes K. The proof is based on the adaptation at the discrete level of the so-called boundedness-by-entropy method, see for instance [2]. I also want to discuss the convergence of the scheme and the long time behavior of the finite-volume solutions of (1). Finally, I will conclude with some numerical experiments.

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References

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