Hilbert schemes, moduli spaces, and symplectic varieties

From May 31 to June 2, 2023

on the occasion of

Manfred Lehn's 6oth birthday

Organizers:

Daniel Huybrechts

Christian Lehn

Christoph Sorger



Speakers

Nick Addington (Oregon) Samuel Boissière (Poitiers) Lothar Göttsche (Trieste) Emanuele Macrì (Orsay)

Sergey Mozgovoy

Dublin)

Kieran O'Grady (Rome)

Michael Rapoport (Bonn)

Alessandra Sarti (Poitiers)

Paolo Stellari (Milano)

Duco van Straten (Mainz)

Claire Voisin (Paris)

Photo: Christoph Sorger

Nantes Université Amphithéâtre LS2N, salles A, B, C - bâtiment 34 2 rue de la Houssinière BP 92208 F-44322 Nantes cedex 3















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Organizers

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Secretariat

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Program

Wednesday, May 31st

10h00	Registration, Welcome coffee
10h45	Opening
11h00-12h00	Claire Voisin: Complete intersections of two quadrics and Lagrangian fibrations
14h00-15h00	Lothar Göttsche: (Refined) Verlinde and Segre formulas for Hilbert schemes of points
15h00-15h30	Coffee
15h30-16h30	Alessandra Sarti: On the cone conjecture for Enriques Manifolds
16h45-17h45	Michael Rapoport: G-Bundles on curves with a twist

Thursday, June 1st

9h30-10h30	Emanuele Macri: Antisymplectic involutions on projective hyper-Kähler manifolds	
10h30-11h00	Coffee	
11h00-12h00	Sergey Mozgovoy: Stability conditions on surfaces	
14h00-15h00	Kieran O'Grady: High dimensional moduli spaces of (semi)stable sheaves on polarized HK varieties of type K3^{[2]} with general moduli	
15h00-15h30	Coffee	
15h30-16h30	Nick Addington: Birational invariants from higher algebraic K-theory?	

19h00 Conference dinner at the restaurant Le 1 https://www.leun.fr/ see map 3

Friday, June 2nd

9h30-10h30	Paolo Stellari: Deformations of stability conditions with applications to Hilbert schemes of points and very general abelian varieties
10h30-11h00	Coffee
11h00-12h00	Samuel Boissière: The Fano variety of lines of a cuspidal cyclic cubic fourfold
13h30-14h30	Duco van Straten: Paramodular forms and Calabi-Yau geometry

Titles and abstracts

Wednesday, May 31st

11h00 -12h00

Claire Voisin (IMJ-PRG) Complete intersections of two quadrics and Lagrangian fibrations

We show that, on the cotangent bundle of a n-dimensional smooth complete intersection X of two quadrics, there are n quadratic functions (which are sections of Sym^2T_X) providing a Lagrangian fibration whose fibers can be described as quotients of Jacobians of hyperelliptic curves by a group of translations of order 2. For n=3, X is a moduli space of rank 2 bundles and the fibration is the Hitchin fibration.

This is joint work with Beauville, Etesse, Höring and Liu.

14h00 -15h00

Lothar Göttsche (ICTP Trieste) (Refined) Verlinde and Segre formulas for Hilbert schemes of points

This is joint work with Anton Mellit. Segre and Verlinde numbers of Hilbert schemes of points have been studied for a long time. The Segre numbers are evaluations of top Chern and Segre classes of so-called tautological bundles on Hilbert schemes of points. The Verlinde numbers are the holomorphic Euler characteristics of line bundles on these Hilbert schemes. We give the generating functions for the Segre and Verlinde numbers of Hilbert schemes of points. The formula is proven for surfaces with $K_S^2=0$, and conjectured in general. Without restriction on K_S^2 we prove the conjectured Verlinde-Segre correspondence relating Segre and Verlinde numbers of Hilbert schemes. Finally we find a generating function for finer invariants, which specialize to both the Segre and Verlinde numbers, giving some kind of explanation of the Verlinde-Segre correspondence.

15h30 -16h30

Alessandra Sarti (Université de Poitiers) On the cone conjecture for Enriques Manifolds

Enriques manifolds are non simply connected manifolds whose universal cover is irreducible holomorphic symplectic, and as such they are natural generalizations of Enriques surfaces. The goal of the talk is to prove the Morrison-Kawamata cone conjecture for such manifolds when the degree of the cover is prime using the analogous result (established by Amerik-Verbitsky) for their universal cover. If time permits I will also show the cone conjecture for the known examples having non-prime degree. This is a joint work with Gianluca Pacienza.

16h45 - 17h45

Michael Rapoport (Universität Bonn) G-Bundles on curves with a twist

In a Bourbaki talk in the 50's, Grothendieck showed that a G-bundle on a complex curve can be described by an equivariant G-bundle on a Galois cover of the curve. He pointed out that not every equivariant G-bundle is obtained in this way if there are ramification points, and posed the problem of describing equivariant G-bundles in terms of objects on the base curve. In the talk I will show that the theory of Bruhat-Tits group schemes allows one to give a satisfactory answer to this problem. Joint work with G. Pappas.

09h30 -10h30

Emanuele Macrì (Université Paris-Saclay) Antisymplectic involutions on projective hyper-Kähler manifolds

An involution of a projective hyper-Kähler manifold is called antisymplectic if it acts as (-1) on the space of global holomorphic 2-forms. I will present joint work with Laure Flapan, Kieran O'Grady, and Giulia Saccà on antisymplectic involutions associated to polarizations of degree 2. We study the number of connected components of the fixed loci and their geometry. In particular, if the divisibility of the ample class is 2, one connected component of the fixed locus is a Fano manifold of index 3, thus generalizing the case of cubic fourfolds. Still in the case of cubic fourfolds, the second component is of general type, thus answering a question by Manfred Lehn.

11h00 -12h00

Sergey Mozgovoy (Trinity College Dublin) Stability conditions on surfaces

In this talk I will discuss Bridgeland stability conditions on the derived categories of surfaces. I will introduce the global dimension of a stability condition, explain its relevance in establishing wall-crossing formulas for the refined DT invariants, and describe the global dimension of a large family of stability conditions on weak Fano surfaces.

14h00 -15h00

Kieran O'Grady (La Sapienza, Rome) High dimensional moduli spaces of (semi)stable sheaves on polarized HK varieties of type K3^{[2]} with general moduli

We will exhibit irreducible components of moduli spaces of (semi)stable sheaves on polarized HK varieties of type K3^{[2]} with general moduli which are HK varieties of type K3^{[2]}, or birational to HK varieties of type K3^{[m]} with arbitrarily high m, or deformations of birational models of singular symplectic varieties. Our work was motivated by examples given by Enrico Fatighenti.

15h30 -16h30

Nick Addington (University of Oregon) Birational invariants from higher algebraic K-theory?

Many have tried to adapt Clemens and Griffiths's approach to irrationality of cubic threefolds to higher dimensions, using different invariants in place of H^3(X,Z): the transcendental part of H^4, derived categories, quantum cohomology... I will report on my attempt to use higher algebraic K-theory, which turns out to be strictly weaker than what Voisin and Colliot-Thélène have already gotten from Bloch-Ogus theory, but (I think) in an interesting way. For a positive result, I can show that the higher K-theory of Kuznetsov's K3 category for a cubic or GM 4-fold looks the same as that of an honest K3 surface.

09h30 -10h30

Paolo Stellari (Università di Milano) Deformations of stability conditions with applications to Hilbert schemes of points and very general abelian varieties

The construction of stability conditions on the bounded derived category of coherent sheaves on smooth projective varieties is notoriously a difficult problem, especially when the canonical bundle is trivial. In this talk, I will illustrate a new and very effective technique based on deformations. A key ingredient is a general result about deformations of bounded t-structures (and with some additional and mild assumptions). Two remarkable applications are the construction of stability conditions for very general abelian varieties in any dimension and for some irreducible holomorphic symplectic manifolds, again in all possible dimensions. This is joint work with C. Li, E. Macrì and X. Zhao.

11h00 -12h00

Samuel Boissière (Université de Poitiers)
The Fano variety of lines of a cuspidal cyclic cubic fourfold

In the framework of the compactification of the moduli spaces of prime order non-symplectic automorphisms of irreducible holomorphic symplectic manifolds, a key question is to understand the geometry of limit automorphisms. Starting from a nodal degeneration of cubic threefold, the general member of the family of Fano varieties of lines of the triple covering branched over the cubic is an IHS manifold equiped with the automorphism induced by the covering. It degenerates to a variety whose singular locus is a K3 surface.

I will present recent results obtained in collaboration with Chiara Camere and Alessandra Sarti that explain how the geometry of this K3 surface permits to define a limit automorphism in a suitable moduli space parametrizing pairs of IHS manifolds with automorphism.

13h30 -14h30

Duco van Straten (Universität Mainz) Paramodular forms and Calabi-Yau geometry

In the talk I describe how a specific Siegel paramodular form has a (conjectural) incarnation in a Calabi-Yau threefold that arose from mirror symmetry in a Grassmannian variety. This insight leads to an unexpected congruence of this form with hilbert-modular form. This is joint work with Vasily Golyshev.

List of participants

Nick	Addington	University of Oregon
Bojko	Arkadij	ETH Zürich
Asher	Auel	Dartmouth College
Chenyu	Bai	IMJ-PRG
Thilo	Baumann	Université du Luxembourg
Tanja	Becker	Universität Mainz
Pietro	Beri	IMJ-PRG
Marcello	Bernardara	Université de Toulouse
Philip	Boalch	IMJ-PRG
Samuel	Boissière	Université de Poitiers
Lukas	Buhr	Universität Mainz
Chiara	Camere	Università di Milano
Jonas	Ehrhard	Universität Mainz
Barbara	Fantechi	SISSA, Trieste
Dino	Festi	Università di Milano
Franco	Giovenzana	TU Chemnitz
Luca	Giovenzana	Loughborough University
Lothar	Göttsche	ICTP Trieste
Michele	Graffeo	Politecnico di Milano
Annalisa	Grossi	Université Paris-Saclay
Kacper	Grzelakowski	University of Łódź

Cynthia	Hog-Angeloni	Universität Mainz
Gaofeng	Huang	Universität Bern
Yifeng	Huang	University of British Columbia
Daniel	Huybrechts	Universität Bonn
Christian	Lehn	Universität Mainz
Manfred	Lehn	TU Chemnitz
Lucas	Li Bassi	Université de Poitiers
Shengxuan	Liu	University of Warwick
Emanuele	Macrì	Université Paris-Saclay
Luigi	Martinelli	Université Paris-Saclay
Simen	Moe	Imperial College London
Christophe	Mourougane	Université de Rennes
Sergey	Mozgovoy	Trinity College Dublin
Yasunari	Nagai	Waseda University
Erik	Nikolov	Universität Hannover
Kieran	O'Grady	La Sapienza, Rome
Patrick	Omukuba	Universität Mainz
Arvid	Perego	Università Genova
Michael	Rapoport	Universität Bonn
Ángel David	Ríos Ortiz	Université Paris-Saclay
Leonardo	Roa Leguizamón	Los Andes, Bogotà

Alessandra	Sarti	Université de Poitiers
Timo	Schürg	TH Bingen
Christoph	Sorger	Nantes Université
Giovanny	Soto	Nantes Université
Paolo	Stellari	Università di Milano
John Olson	Torger	University of Oslo
Duco	van Straten	Universität Mainz
Mauro	Varesco	Universität Bonn
Claire	Voisin	IMJ-PRG
Xiaohan	Yan	IMJ-PRG
Shizhuo	Zhang	MPI Bonn

Coming to Nantes Université

FROM DOWNTOWN: see tramway network on map 1

At the stop «Commerce», take the tramway line 2 in the direction of «Orvault Grand Val». Get off at the stop «Michelet-Sciences».

The cost of one-hour-valid ticket is $1,70 \in$. There are some vending machines at the stop «Commerce». You can also buy a book of 10 tickets which costs $16 \in$. Breakdown ticket available from the driver on board the bus for $2 \in$.

FROM THE TRAIN STATION: see transport map on map 1

Upon your arrival at the main station, take the North exit (Sortie nord) and walk in the direction of the tramway stop «Gare Nord» which is in front of the main entrance of the station. Take the tramway line 1 in the direction of «François Mitterrand», get off at «Commerce». Take the tramway line 2 in the direction of «Orvault Grand Val» and get off at «Michelet Sciences». **ATTENTION**: The vending machines may not take non-France issued credit/Banking cards.

For detailed bus schedules and maps please visit TAN http://www.tan.fr

FROM THE AIRPORT:

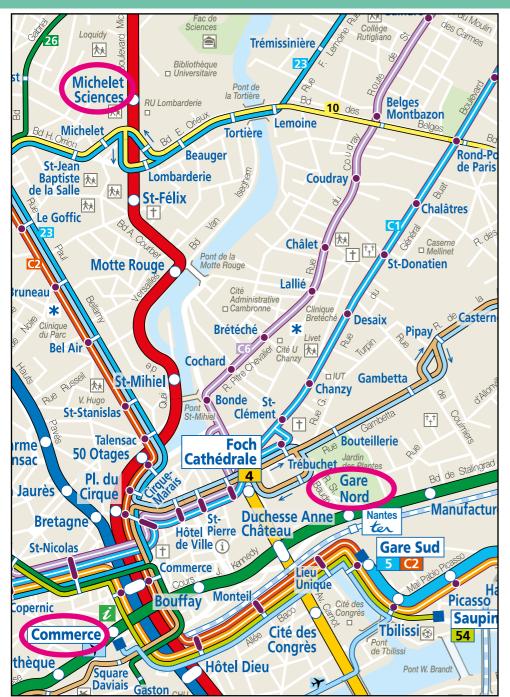
Get to the city center by the airport shuttle bus (navette aéroport) in 20 minutes. The final stop of the shuttle is «Commerce». There is one bus every 30 minutes. From there, take the bus line as indicated in the paragraph «FROM DOWNTOWN».

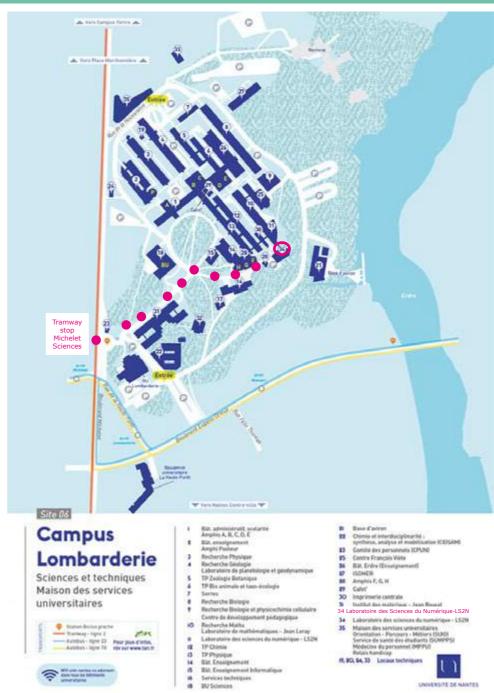
Website airport: https://www.nantes.aeroport.fr/fr

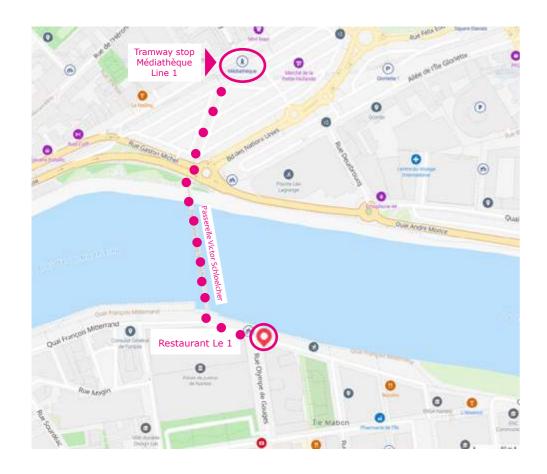
By taxi: The station head is located just in front of the terminal opposite Arrivals 1. The cost is around 40€.

Once on the campus: Walk to building 34 (Laboratoire des Sciences du Numérique LS2N) where the conference will take place. see map 2









Restaurant Le 1 1 rue Olympe de Gouges 44200 Nantes tél. 02 40 08 28 00 https://www.leun.fr/

• • • walking distance (5 minutes)

