

**Schedule of the workshop :**  
**"Recent Developments in Conformal Geometry "**  
**Nantes : October 22-26, 2012**

	Monday 22	Tuesday 23	Wednesday 24	Thursday 25	Friday 26
9h30-10h30		V. Bour	Z. Djadli	M. Herzlich	M. Fischmann
10h30-11h00		<i>Coffee break</i>			
11h-12h		A. Neves	B. Ammann	M. Eastwood	V. Minerbe
12h30-15h00	14h-15h00 Welcome	<i>Lunch break</i>			
15h00-16h00	K. Hirachi	C. Aldana	M.M. Gonzales	C. Rossi	
16h00-16h30	<i>Coffee break</i>				
16h30-17h30	C. Guillarmou	F. Madani	N. Grosse		

### Titles and Abstracts

**Clara Aldana (Albert Einstein Institut, Germany)**

*Conformal surgery and compactness of relatively isospectral sets of surfaces.*

*Abstract :* We consider surfaces that have boundaries and ends that are asymptotic to cusps or asymptotic to funnels. We define the concept of being relatively isospectral. I will explain how we prove compactness of relatively isospectral sets using conformal surgeries. The results to be presented in the talk are joint work with Pierre Albin and Frederic Rochon.

**B. Ammann (Universität Regensburg, Germany)**

*Lower bounds for the Yamabe invariant*

*Abstract :* In this talk I want to give an overview over some old and new results about the (smooth) Yamabe invariant in joint work with M. Dahl and E. Humbert. We will see for example that the Yamabe invariant of a simply connected compact spin manifold of dimension 5 is between 45 and 79. In dimension 6 it is between 49.9 and 96.3. Similar estimates hold for 2-connected compact spin manifolds with vanishing index in higher dimensions.

**V. Bour (Université Joseph Fourier, France)**

*Rigidity results for manifolds with positive Yamabe constant*

*Abstract :* On a Riemannian manifold, the positivity of the Yamabe constant induces a Sobolev inequality. If we combine this Sobolev inequality with some Weitzenböck identity,

we obtain a rigidity result for elements satisfying an elliptic condition. For instance, if we use the Bochner-Weitzenböck formula, we obtain a vanishing result for harmonic forms. In the talk, we will present a number of pinching results on compact and non-compact manifolds, which are obtained by this method.

**Z. Djadli (Université Joseph Fourier, France)**

*Four order critical geometric flows*

*Abstract :* we will discuss some work in progress concerning the analysis of some critical fourth order flows. For these flows, the short time existence of solution is not an easy issue as the DeTurck's trick doesn't work and we will present a strategy in order to prove short time existence.

**M. Eastwood (Australian National University, Australia)**

*Conformally Fedosov manifolds*

*Abstract :* Symplectic and projective structures may be compatibly combined. The resulting structure closely resembles conformal geometry and a manifold endowed with such a structure is called conformally Fedosov. This talk will present the basic theory of conformally Fedosov geometry and, in particular, construct a Cartan connection for them. This is joint work with Jan Slovák.

**M. Fischmann (Humboldt Universität, Germany)**

*Further structures of the conformal powers of the Dirac operator.*

*Abstract :* The Dirac operator on a semi Riemannian spin manifold is metric dependent. A main property of the Dirac operator is its behavior under conformal change of the metric, i.e. it is a conformal covariant of certain bi-degree. Now the question arise, how the powers of the Dirac operator do transform under conformal rescaling ? For odd powers there exist some curvature correction terms which one has to add to obtain a conformal covariant operator of a bi-degree depending on its power. Only few examples are known due to the complexity of the underlying algorithm. After a short introduction of the conformal powers of the Dirac operator I will focus in my talk on the structure of these operators for low powers. It turns out that there exist first order operators which can be combined to the desired operator. Thus one has to analyse the structure of these first order operators and how one has to combine them to obtain the conformal powers of the Dirac operator.

**M.M. Gonzales (Universitat Politècnica de Catalunya, Spain)**

*Fractional order operators in conformal geometry*

*Abstract :* In this talk we will have a closer look at the relations between scattering operators of asymptotically hyperbolic metrics and Dirichlet-to-Neumann operators for uniformly degenerate elliptic boundary value problems. Joint with J. Qing, we consider fractional Yamabe problems, that include the boundary Yamabe problem formulated by Escobar. We observe an interesting Hopf type maximum principle together with interplays between analysis of weighted trace Sobolev inequalities and the conformal structure of the underlying manifolds, which extend the phenomena displayed in the classic Yamabe problem. In the last part of the talk, I will introduce an ongoing project on the complex/CR version.

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**C. Guillarmou (École Normale Supérieure de Paris, France)**

*On the renormalized volume in odd dimension*

*Abstract :* We discuss some properties of the renormalized volume for odd dimensional Poincaré-Einstein manifolds, and the relation with certain equations appearing in conformal geometry (like  $\sigma_k(\text{Schouten}) = \text{constant}$ ). This is joint work with S.Moroianu and J-M. Schlenker.

**N. Grosse (Universität Leipzig, Germany)**

*The Yamabe equation on special types of open manifolds*

*Abstract :* The Yamabe constant is a conformal invariant of a Riemannian manifold given by a variational problem that was studied a lot during the last decades. Solutions of the corresponding Euler-Lagrange equation, the Yamabe equation, on closed manifolds always exist which implies that in each conformal class there is a metric with constant scalar curvature. We want to study the existence of such solutions on open manifolds which is quite hard for arbitrarily open manifolds since the method that proves existence in the closed case highly relies on the existence of compact Sobolev embeddings which in general do not exist on open manifolds. That is why we restrict to two special classes. First we can study manifolds of bounded geometry with positive Yamabe constant where weighted Sobolev embeddings are used to analyze certain weighted versions of the Yamabe constant which gives the desired solution in the limit. Moreover, we can deal with complete Riemannian manifolds of finite volume and positive Yamabe constant where we use the description of the Yamabe constant as a limit of conformal eigenvalues of the so-called Yamabe operator. This method also gives a different proof for the closed manifolds with positive Yamabe constant.

**M. Herzlich (Université de Montpellier, France)**

*Conformal geodesics in parabolic geometries*

*Abstract :* We give a simple characterization of the parabolic geodesics introduced by Cap, Slovak and Zadnik for all parabolic geometries.

**K. Hirachi (Tokyo University, Japan)**

*Ambient metric for even dimensional conformal structures*

*Abstract :* The ambient metric of Fefferman and Graham is now a standard tool of conformal geometry. It is uniquely determined if the dimension is odd, but has ambiguity for even dimensions. In this talk, I describe the ambiguity precisely and formulate the jet isomorphism theorem of conformal structures. This is joint work with Robin Graham.

**F. Madani (Universität Regensburg, Germany)**

*Conformal-harmonic maps.*

*Abstract :* Conformal harmonic maps are defined from an even dimensional conformal manifold to a Riemannian manifold which satisfy a certain conformally invariant fourth order equation. They can be viewed as a natural extension of harmonic maps defined from a surface to a manifold. In this talk, we start by a short introduction to harmonic maps and existence theorems. We give a definition of C-harmonic maps and prove an existence theorem, analogous to the Eells-Sampson one.

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**V. Minerbe (Université Pierre et Marie Curie, France)**

*The Ricci flow turns bolts into nuts.*

*Abstract :* In this talk, we will describe some examples of unstable fixed points of the Ricci flow in dimension four and study how the Ricci flow affects a small perturbation of them. The aim is to confirm a conjecture stemming from physicist numerical simulations due to Headrick-Wiseman and Holzegel-Schmelzer-Warnick.

**A. Neves (Imperial College, UK)**

*Min-max theory and Willmore conjecture*

*Abstract :* I will explain how one can use the min-max theory for minimal surfaces to solve the Willmore conjecture. This is joint work with Fernando Marques.

**C. Rossi (Université d'Avignon, France)**

*Globally Hyperbolic Conformally Flat Maximal Spacetimes*

*Abstract :* A conformally flat space-time  $M$  is a  $(G, X)$ -manifold where  $X$  is the universal Einstein space-time  $\widetilde{\text{Ein}}_{1,n}$  ( which is  $\mathbb{S}^n \times \mathbb{R}$  with the conformal class of the metric  $ds^2 - dt^2$ , where  $ds^2$  is the standard metric on the sphere and  $dt^2$  on  $\mathbb{R}$ ) and  $G$  is the identity component of his group of conformal diffeomorphisms. It turns out that this group is the universal cover of  $\text{SO}(2, n)$ , where  $n$  is the dimension of  $M$ . The causal structure of a lorentzian manifold is conformally invariant, so we have a well defined causal structure on  $M$ . We assume that this causal structure is globally hyperbolic. With this hypothesis when  $M$  is  $C_0$ -maximal (which means that every conformal embedding of  $M$  in a globally hyperbolic conformally flat space-time sending Cauchy hypersurfaces to Cauchy hypersurfaces is surjective) we can proof some results about the domain of injectivity of the developping map. In particular we will show that : If  $M$  is a globally hyperbolic conformally flat  $C_0$ -maximal space-time and if there are two conjugates points  $p, q$  in the universal cover of  $M$  then  $M$  is a finite quotient of the Einstein space-time .