Jean Leray

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Jean Leray, who died on November 10, 1998 — three days after his 92nd birthday, was a major figure of XXth century mathematics. Between 1931, when his first research announcement (in fluid mechanics) was published, and 1994, when his last paper (in complex analysis) appeared, his complete list of publications contains 132 entries. Leray's collected works (in fact a *Selecta*) [Leray], published shortly before his death by Springer Verlag and the Société mathématique de France[1], consists of a three volume set of 1,600 pages, containing 53 papers mostly written in French.

Leray worked in three different fields : Topology, Partial differential equations and Complex analysis. He considered himself as an applied mathematician, and it is no easy paradox that a large part of his work should be in pure mathematics.

When Leray was a student at Ecole normale supérieure in Paris, between 1926 and 1929, the mathematics faculty at the University of Paris was not at its best : long gone were the prewar days when French analysts were inventing modern integration, when Henri Poincaré was, with Hilbert, the foremost mathematician in Europe. Poincaré died prematurely in 1912 ; many others[2] turned away from mathematics, became involved in politics or administration. In the twenties, only two senior Paris mathematicians stood out : Jacques Hadamard and Elie Cartan. With very few exceptions, the intermediate generation had been wiped out[3] by the war.

Leray and his fellow students at Ecole normale supérieure were a brilliant group, including such men as André Weil, Henri Cartan, Jean Dieudonné, Claude Chevalley, Charles Ehresmann, Jacques Herbrand… But whereas the latter, true heirs at least in that respect to the strongly biased attitudes prevalent in France[4] against applied mathematics, were interested in pure mathematics, and found a major source of inspiration in Hilbert's formalism and the German school, Jean Leray, who had a strong interest in physics, leaned towards towards the applied aspects and felt more in tune with the intellectual resources available in Paris ; all of them, however shared an endurable admiration for Elie Cartan[5] : « Cartan's work proved to be fundamental and his teaching, constantly renewed, was shining with all its brilliance[6] ». Leray started working in fluid mechanics and wrote his thesis on that topic under Henri Villat[7] in 1933. The philosophical difference between André Weil and his friends (who would create the influential Bourbaki group in the thirties), and Jean Leray would prove to be long lasting.

Leray's thesis and first papers on partial differential equations

The mathematical difficulties of fluid mechanics are formidable. Although the equations for an

[1] Leray refused until the mid nineties to yield to the custom of publishing collected works of scientists while they are still alive and active, arguing that he was still working.

[2] Among others : Borel, Painlevé and Picard.

[3] See [Andler 1994] and [Andler 2005].

[4] Not only in France, of course ; Oxford's George Hardy was no less prejudiced against applied mathematics.

[5] Elie Cartan was Henri's father.

[6] Letter to Pierre Lamandé, October 1990, kindly shown to us by Jean Leray's daughter Fraçois Pecker.

[7] Henri Villat was a « mechanician », a professor of fluid mechanics at the university of Paris; he became a member of the French Académie des sciences in 1932. In that powerful position he provided, again and again, a considerable support for Leray at the beginning of his carreer. Leray remained very close to Villat until his death in 1972.

incompressible fluid[8], the so-called Navier-Stokes equations, have been known since the XIXth century, very little progress had been made on solving them before Leray. For such equations, several questions immediately arise :

-- can one prove mathematically the existence and possibly uniqueness of solutions (under appropriate conditions on the data) ?

-- if indeed a solution exists, will it exist only for a short period of time, or forever (concretely, the physical object might « explode » after a certain amount of time) ?

-- what exactly does one mean by a solution ? Is it a smooth function (if so how smooth) ? If one simply looks at the flow of water in a river, or at the wake of a boat on that river, it becomes obvious that the solutions ought to be very irregular (turbulent) -- even with some discontinuities. But then how can one speak of the partial differential equation in the first place ?

As Peter Lax in his forword to volume II of [Leray] mentions : « Physicists sometimes deride such existential pursuits by mathematicians, saying that they stop when things are getting interesting ». But Lax continues by pointing out that « what Leray found out […] was far more interesting for the physics of fluids than anything thought before ».

Leray's dissertation concerned time stationary problems (time independent) [Leray 1933c][9]. He used a generalisation of a method of Erhard Schmidt for solving non linear integral equations, used it to study some elliptic partial differential equations and the Navier-Stokes equation in dimension 2 and 3.

The following year, two major papers by Leray appeared[10] – « Sur le mouvement d'un fluide visqueux emplissant l'espace[11] » [1934b] and « Topologie et analyse fonctionnelle[12] » [1934c] in collaboration with Schauder. In some ways, those papers extended his dissertation, but they brought in some completely new ideas.

The Polish mathematician Julius Schauder[13], seven years older than Leray, was a student of Steinhaus in Lvov in the early twenties. With S. Banach, Steinhaus[14] had created a lively mathematics department in Lvov, where they set the foundations of modern functional analysis. Until then, functions were only considered as complicated individuals, given by complicated formulas or through properties of their graphs. What the new ideas made possible was to consider functions as points of some infinite dimensional space, and to use the geometry and the topology of those spaces. A tremendous change of perspective !

In some cases, a function from a space to itself has fixed points ; this has to do with the « topology » (the shape) of the espace. For instance, Brouwer proved in 1911 that a smooth function from the unit disk to itself wich leaves the unit circle fixed has a fixed point in the unit disk[15]. In 1930, Schauder proved a similar result in an infinite dimensional setting, where it could be applied to spaces of functions.

Schauder received a Rockefeller scholarship which allowed him to visit Paris in May of 1933[16]. There Hans Lewy, a prominent mathematician from Göttingen who had fled Germany after Hitler's rise to

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[8] These equations are partial differential equations (pde), *ie* equations involving the function and some of its partial derivatives. Many problems in physics (heat equation, wave equation, Maxwell's equations, Navier-Stokes equations , Einstein's equations) are pde.

[9] The references to Leray's papers in this Note are numbered in a consistent fashion with the complete bibliography available on ???

[10] The words used now to describ them are « epoch making », « landmark »

[11] On the movement of a space filling visquous fluid.

[12] Topology and functional analysis.

[13] See the biographical note on Schauder : http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Schauder.html

[14] Steinhaus wrote his thesis in Göttingen with D. Hilbert.

[15] Solving an equation f(u) = 0 can be seen as finding a fixed point u for F(u) = u + f(u).

[16] The purpose of the visit was to work with Jacques Hadamard, a formidable figure of French mathematics, who had a seminar which played an important role in bringing new ideas and foreign mathematicians to Paris.

power, introduced him to Leray[17]. Leray had been advised by Leon Lichtenstein and others during the previous winter which he spent in Germany to read Schauder's papers. It had convinced him that Schauder's topological methods were a powerful tool to establish existence properties of solutions of partial differential equations independently of uniqueness. The two young men worked together. In a matter of a few weeks they generalised to the infinite dimensional case the notion of degree of a map, a more refined topological invariant than the fixed point, and saw how the notion could be applied to partial differential equations. The paper was quickly written and accepted for publication in Annales de l'Ecole normale supérieure[18]. The « Leray-Schauder method » was to become a central tool in analysis, a remarkable application of (algebraic) topology to analysis.

The paper in Acta mathematica studies the Navier-Stokes equation (see [Chemin] for a detailed description). Leray proves that regular solutions for the « Navier » equation[19] exist up to a time T and characterizes T. He introduces the notion of a *weak* solution (and in order to do this he defines what is now called a *Sobolev space*), giving a precise meaning to an « irregular » solution of the equation and proves that there exist such weak solutions (an irregular solution corresponds to the physical notion of turbulence). A "strong-weak" uniqueness theorem then shows that, when there are both solutions in the usual sense and weak solutions, the two should coïncide.

The importance of the Acta paper is threefold : on the one hand, the notion of a weak solution is a paving the way for two essential tools of post world war II analysis : the « Sobolev » spaces and Schwartz's theory of distributions. On the other hand, it provides a huge leap forward in understanding the behaviour of solutions of a specific non-linear partial differential equation. Finally, it gives weight to the idea that Navier-Stokes equation does give a good model for fluid mechanics.

As it turns out, progress on understanding solutions of the Navier-Stokes equation have been few. After Leray's 1934 paper, the next substantial developement came in 1951 with E. Hopf's work, later generalized by Ladyzhenskaya. In spite of substantial efforts by several mathematicians over the last fifty years, a complete solution of the Navier Stokes equations is still today an open problem[20].

The war years and the papers in algebraic topology

Less than at any other periods, could politics be left aside. Leray was in Germany when the Reichstag burned. It is in Paris that he met Schauder because Göttingen was not as attractive as before, after several Jewish faculty members had fled Germany. Schauder himself was a Jew ; his situation in Poland in the thirties under the Pilsudski regime was difficult. But the worst was yet to come : he was arrested by the Gestapo in Lvov shortly after the beginning of the war between Germany and the Soviet Union and was never again seen alive.

As most university graduates in France, Leray was a reserve officer. When the war broke out in September of 1939, he was called to active duty. After France's military collapse in May-June of 1940 and the subsequent infamous armistice that the French government signed on June 17, Leray became a prisoner of war and was sent to a prisoner's camp in Austria (Oflag XVII A) on July 2nd. He remained there during almost five years, until the camp was liberated on May 10 of 1945.

The prisoners in the camp were mostly educated men, career or reserve officers ; many of them were still students. As in several other camps, a "university" was created and Leray became its rector. Classes were taught, exams were given, and degrees granted, with some degree of recognition by French authorities of the time. As for research, to fight the feeling that he might be losing the best productive years of his life, Leray wanted to resume his work. But he was confronted with a dilemma. If he continued working in fluid mechanics, he might be forced to collaborate with the German war effort. Instead, he decided to pursue some ideas in algebraic topology (a « useless » pursuit) that he had foreseen during his collaboration with Schauder, using some inspiration from differential

[17] Leray recounted his relationship with Schauder in [1979].

[18] The authors dedicated their paper to Leon Lichtenstein, who had passed away during the summer of 1933, his health having quickly deteriorated due in good part to the political situation in Germany. But the editor of the journal, Emile Picard, removed the dedication ; instead he added several references to his own papers !

[19] Leray calls Navier equation what is now usually called Navier-Stokes equation.

[20] It is one of the seven problems for which the Clay mathematical foundation has set a reward of 1 million dollars for its solution (see http://www.claymath.org).

geometry, a field that he knew well from Elie Cartan[21].

The basic idea of algebraic topology is to associate algebraic objects to « topological » spaces, where topological refers to the science of shapes and their deformations. For instance, there is a big difference between a disk and a disk punctured at its center : if one draws a curve, however complicated, on a disk, one can always deform it to a point. On the other hand for the punctured disk, that same curve cannot be deformed to a point if it loops around the hole. The simplest algebraic object associated to a space is its fundamental group : for a disk, the fundamental group has one element : 0, whereas the fundamental group of the punctured disk is the set **Z** of all positive or negative integers (the integer corresponds top the number of loops around the hole, the sign whether one turns clock or counterclockwise. The difference between 0 and **Z** encodes algebraically the topological difference coming from the puncture. There are many other algebraic objects corresponding to topological or geometric spaces, the main ones being homology and cohomology groups. A familar example : the « curl » condition for a vector field to derive from a scalar potential has to do with cohomology.

In the course of his work in Oflag XVII, with very little access to printed material and no contacts with other mathematicians, Leray introduced two fundamentally new notions which played an essential role in geometry and topology since the late fourties : the theory of sheaves and spectral sequences. Spectral sequences are a powerful tool for computing cohomology groups by successive approximations. They encode extremely complex informations in a tractable way. Sheaves are a tool to establish global properties of spaces from studying their local properties[22]. Quoting M. Atiyah[23] : « [Sheaf Theory] has proved to be a tool of enormous versatilty applicable in almost any situation where the situation between local and global makes sense. »While still a prisoner, Leray submitted his first results, four research announcements, to the Académie des sciences through Villat ; they were promptly published in the *Comptes rendus de l'Académie des sciences* in 1942 [1942a, b, c, d]. Villat helped his former student in a major way two more times before the end of the war : in 1943, by arranging for his appointment as « Maître de conférences » (associate professor) at the Paris university, though of course he could not teach in Paris ; in 1944, by nominating him for an associate membership (membre correspondant) of the French Academy of sciences.

In 1945, three papers were published [1945a, b, c] in the Journal de mathématiques pures et appliquées whose editor was Villat, followed by several others between 1946 and 1950. While the 1945 papers contained the necessary foundations, it is in two research announcements published in 1946 : [1946a, b] that the ground breaking notions of a sheaf and of a spectral sequence appeared for the first time. Contrary to what had happened in 1933-1934, when the importance of Leray's papers had been immediately acknowledged, it took a while for the new concepts to be understood[24].

The concepts involved were radically new, the exposition hard to follow. In the United States, for instance « Most people [...] found Leray's papers obscure »[25]. Leray himself was not a charismatic lecturer, to say the least, nor was he interested in being one. One had to be very determined and persistent to follow[26]. Ironically, it is through mathematicians belonging to Bourbaki that Leray's ideas in algebraic topology became known and universally used. Weil mentions in his Collected works a conversation he had with Leray in June of 1945[27], during which Leray outlined some of his ideas. Weil immediately realised their importance[28]. In turn, related the conversation to Henri Cartan. It is Cartan and some young mathematicians or students close to Cartan, notably Jean-Louis Koszul,

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[21] Leray wrote up the lecture notes of one of Cartan's courses, published in 1935 : *La méthode du repère mobile, la théorie des groupes continus et les espaces généralisés.*

[22] A very simple example of the distinction : locally, a sphere is in most respects like a plane. On the other hand, globally, a sphere is bounded, whereas a plane is not.

[23] Jean Leray's citation, Royal Society, 1983.

[24] Fortunately, the history of algebraic topology, in particular Leray's contribution, has been studied quite extensively : see Borel's introduction to volume I of [Leray], Miller's paper in [Kantor], [James].

[25] G. Whitehead, *Letter to John McCleary*, 1997, quoted in Miller's paper in [Kantor].

[26] Later, when he taught at Collège de France, having an audience of eight attendants suited him well ; when he occasionally had nine, he worried that his lectures might be too easy…

[27] At the time, the relationship between the two was particularly tense. Weil, who was of Jewish descent, had fled France in 1939 to avoid the draft, and eventually spent the war in the United States (see Weil's Memoirs : *The apprenticeship of a mathematician,* Birhaüser, 1992), while Leray was a prisoner in Austria.

[28] He understood enough to give a new proof of the De Rham theorem.

Jean-Pierre Serre and Armand Borel[29], who gave the theory its final form and its most spectacular applications.

It is a mild understatement that Leray's ideas, reworked and refined by Cartan, Koszul, Serre, Borel... revolutionised large parts of pure mathematics after World War II. In the process of algebraization of mathematics, which is one of the main features of XXth century mathematics, sheafs and spectral sequences are the jewel in the crown — tools that are central in Cartan's famous theorems A and B in complex analysis, Serre's computation of the homotopy groups of the *n*-dimensional sphere, Serre and Grothendieck's reformulation of algebraic geometry, Sato's algebraic analysis, etc.

Oflag XVIIA was liberated in May of 1945. In 1947, the Collège de France had to choose Lebesgue's successor for a chair in mathematics. With the strong support of Villat, Leray was chosen. He was a professor there until he retired in 1978. Although he was hired as an analyst, he devoted his first few years to lectures on his work in algebraic topology.

The post war period : partial differential equations and complex analysis

After 1950, Leray returned to analysis, leaving sheaves and spectral sequences in good hands. Before the war, he had studied mostly elliptic and parabolic partial differential equations ; now he turned to hyperbolic problems, like the wave equation, and to partial differential equations in the complex domain.

The generalisation of the Friedrich-Lewy approach, which was effective for the wave equation, to general hyperbolic initial value problems required technical expertise and some topology. Leray of course had both ; his lectures on this topic [1953a] at the Institute in Princeton in 1952 remained unpublished[30], but the mimeographed version circulated a lot. Combined with Gårding's work, they gave a complete solution, provided the hyperbolic equation has simple characteristics. In the sixties, he worked on the multiple characteristics case.

Although the focus of his work turned, in the mid fifities, to multi-variable complex analysis, as we will now see, he came back to partial differential equations, his main concern, time and time again from the fifties to the early nineties. His interest for complex analysis was indirect : he still had partial differential equations in mind, but in the complex domain. It required more knowledge of multivariable complex analysis than was available at the time. Ironically, complex analysis was, at the time one of the domains where Leray's sheaf theory and spectral sequences proved most effective. Work by Henri Cartan, Cartan-Serre, Remmert and others confirmed the power of algebraic methods in complex analysis.

But applications to analysis required constructive methods, with explicit formulas (for example in the computation of Laplace transforms). For Leray's work, the starting point was his 1952 calculation on the elementary solution of hyperbolic equations with constant coefficients, where he built on previous work by Herglotz and Petrowski. He then considered linear equations with variable coefficients, and later proceeded to non-linear equations. This program was implemented in an impressive series of papers published between 1957 and 1964, with subtitles « Problème de Cauchy I, II, …, VI[31] » [1957b, 1958a, 1959b, 1962b, 1964a]. They included generalisations in the severable variables case for the classical Cauchy residue theory and the Cauchy inversion formula. For the latter, Leray proved what is now known as the Cauchy-Fantappiè-Leray formula.

As G. Henkin writes[32] : « Without exaggeration one can say that during the fifties-sixties the ideas of Leray twice radically changed the direction of the development of contemporary complex analysis. The Leray sheaf theory was he main tool for the great breakthrough in complex analysis in the early

[29] In his tribute to Armand Borel, who died in 2003, at the Académie des sciences, Serre recalls that Borel, then a young postdoc from Switzerland, spent the year 1949-1950 in Paris with a CNRS fellowship : « A very good choice (for us as well as for him) : Paris was the place where what the Americans call « French topology » was being created, with Leray's lectures at Collège de France and the Cartan seminar at Ecole normale. Borel was an active participant in the Cartan seminar, and sat on Leray's lectures. He managed to understand the famous « spectral sequence », a far from easy task — and explained it to me so well that I have never stopped using it during the following fifty years. »

[30] Except for a 1984 russian translation, and a short note stating the most important results.

[31] Actually, paper V was never published ; the ideas therein, being partly explained in the others.

[32] Foreword to volume III of [Leray].

fifties. […] Thus in the sixties, thanks to Leray, the constructive methods of residue theory and of integral representations occupied once again a first rank position in the complex analysis of severable variables. »

Leray's influence

It is not surprising that Leray enjoyed a considerable international recognition. He was elected a member of the Académie des sciences in Paris in 1953[33]. He was a member of fourteen foreign academies (including of course the Royal society where he was elected a foreign correspondent in 1983), was invited twice as a lecturer at the International Congress of Mathematicians, was the president of the International Congress of Mathematicians in Nice in 1970 ; among other prestigious prizes, he was the co-recipient (with André Weil !) of the Wolf prize in 1979.

He was frequently invited abroad[34], where he had friends in several countries[35]. His influence in France was felt perhaps more directly through his mathematics than otherwise. Some exceptions need to be pointed out : Leray had relatively few, but remarkable, PhD students, among whom Yvonne Choquet-Bruhat. Although he was not involved in such pursuits himself, he was genuinely interested in numerical analysis. In the late fifties, he played a pivotal role in getting Jacques-Louis Lions, who attended his lectures at Collège de France, to work on non-linear problems[36]. A few years later, Lions would start building a large group of mathematicians and network of institutions specialised in applied mathematics, mostly partial differential equations, in particular non-linear problems and numerical analysis. This relationship explains Peter Lax' description of Leray as the « intellectual guide of the present distinguished school of applied mathematics »[37].

When in the late sixties France, perhaps more than other countries, fell for the introduction of « modern mathematics » in secondary (and even primary) education, this was done in the name and under the indirect influence of Bourbaki[38]. Leray was one of the very few research mathematicians who opposed the reform. He wrote papers, took part in meetings and discussions. One of those papers [1966b] is dedicated to « Mlle A, whose tears I collected ». His view was that teaching mathematics in a way which hid its fruitful connections with science and technology would be an aberration[39].

Jean Leray was a private man — his reluctance to speak about himself is rarely broken, except perhaps in the remarkable book of interviews and pictures of scientists by Marian Schmidt[40]. Son of two elementary school teachers, he married in 1932 Marguerite Trumier, who was the daughter of two colleagues of his parents in the same town of Chantenay, in the West of France. Marguerite worked as a mathematics teacher in high school all her life. They had three children : Jean-Claude in 1933, who became an engineer, Françoise (1947), a research professor in biology and Denis (1949), a doctor.

In almost no respect does Jean Leray fit in predetermined categories. An applied mathematician whose contribution to geometry and topology was accidental, but « more influential than [that of] the experts working in these fields[41] ». A French mathematician who was at the same time part of, and a strong counterweight to, the dominant trends of French mathematics in the Fifties and Sixties, based on two principles : mathematics as a self contained pursuit, where structures played a central role. For Leray, inspiration came from the physical world, from physics, mechanics and technology. Yet he rejected utilitarianism as being counterproductive for science (in no way was he a proponent of « application driven research ») ; in a conference, he wrote : « Man is less attached to living than to create and enjoy beauty » [1972a]. In his view, science is most effective when scientists do not seek applications, but rather knowledge for it's own sake.

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[33] By contrast, the mathematicians belonging to Bourbaki had to wait until the mid-seventies to enter the Académie.

[34] During ten years, he spent the fall semester visiting the Institute for advanced study in Princeton

[35] Among those, Sobolev (Soviet Union), Kotake (Japan), Gårding (Sweden), Oppenheimer and von Neumann (USA).

[36] They wrote a paper together. Also Lions was Leray's successor at Collège de France in 1973.

[37] Foreword to volume II of [Leray].

[38] Although many of the actual members of Bourbaki did not agree with the way the reform was being carried out, they did not take much part in the debate.

[39] Quoted in [Meyer]

[40] [Schmidt]

[41] M. Atiyah, *op. cit.*

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