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Fredholm theory in complex manifolds with complex parameters: analyticity properties and Landau singularities of the resolvent.
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Let Δ be a domain in $\mathbf{C}^q$, M a complex n-dimensional manifold and S a closed subset of $\Delta \times M$ such that S and $D = \Delta \times M \setminus S$ are trivial fiber spaces with base Δ; let S_k and D_k be their fibers above $k \in \Delta$; let $\hat{\Delta}$ be the universal covering space of Δ; denote by $\hat{k}$ any point of $\hat{\Delta}$ projecting onto k; let $h(\hat{k}) \in H_n(D_k)$ be a compact n-dimensional homology class of D_k depending continuously on $\hat{k}$; let $\omega(k, z)$ be a holomorphic $(0, n)$-differential form in $\Delta \times M$ and let $G(k, z, z')$ be an analytic function of $(k, z) \in D$ and $z' \in M$; let $F(\hat{k}, z, z', \lambda)$ be the Fredholm resolvent of the kernel G, that is, the solution of $F(\hat{k}, z, z', \lambda) = G(k, z, z') + \lambda \int_{h(\hat{k})} F(\hat{k}, z, z_1, \lambda) G(k, z_1, z') \omega(k, z)$, Fredholm's fixed integration domain being thus replaced by $h(\hat{k})$.

Fredholm's argument applies; therefore F is a meromorphic function of the parameter $\lambda \in \mathbf{C}$; this function is holomorphic in $\hat{k} \in \hat{\Delta}$, $z \in M \setminus S_k$ and $z' \in M$.

From now on assume $S = S^1 \cup \cdots \cup S^m$, $m < n - 1$, S^i being regular complex analytic hypersurfaces of $\Delta \times M$. The simple critical points a of the stratum $A = S^1 \cap \cdots \cap S^m$ correspond in the fiber to simple or quadratic pinches of the manifolds $S^1, \cdots, S^m$; these points a project onto the Landau variety LA, which is a regular complex analytic hypersurface of Δ; call its local equation $l(k) = 0$. Given a, its projection k_a in LA and some $\hat{k}_a \in \hat{\Delta}$ with projection k_a; then $F(\hat{k}, z, z', \lambda)$, in the neighborhood of $\hat{k}_a$, if considered as a function of k, admits a branching singularity on LA. It is studied under the following assumption: G has a first order polar singularity on each S^i.

If $n - m$ is even, then in a neighborhood of $\hat{k}_a$, $F(\hat{k}, z, z', \lambda) = F_1(k, z, z', \lambda) + l(k)^{(n-m-1)/2} F_2(k, z, z', \lambda)$, where the F_i $(i = 1, 2)$ are meromorphic; thus one has to use in that neighborhood the two-sheeted covering of Δ ramifying on LA.

If $n - m$ is odd, then a less simple result is given; in a neighborhood of $\hat{k}_a$, one has to use the infinite-sheeted covering of Δ ramifying on LA.

Arguments are based on the Picard-Lefschetz formula about vanishing cycles.

The introduction shows how interesting these results and their possible extensions are for quantum field theory.

Reviewed by *Jean Leray*