MR603126 (82d:34061) 34D15 (41A60)
Gingold, H.
A new basis for singularly perturbed problems: a representation theorem.
Adv. in Appl. Math. **1** (1980), *no. 1*, 67–107.

The author provides the following contribution to singular perturbation theory: Let $y(t,u) = \sum_\nu y_\nu(t)u^\nu$ be a holomorphic function of u $(u \in \mathbf{C}, |u| < 1)$ continuously depending on t $(0 \le t \le 1)$; assume $y_\nu(t) = O(\nu^\alpha)$ with $\alpha < \frac{1}{2}$; assume $y(t, \cos\theta)$ to be an integrable function of θ on $0 \le \theta \le \pi$; assume $y(t,u)$ to be continuous on $-1 < u \le 1$; then an expansion of $y(t,u)$ on $-1 < u \le 1$ is given in terms of Čebyšev polynomials; its coefficients are continuous functions of t; using the Cesàro summation process it converges even for $u = 1$.

The case $0 \le t < \infty$ and the behaviour for $t \to \infty$ are also studied. This applies for example to the continuous function $y(t, \varepsilon)$ of t $(0 \le t < \infty)$ and ε $(\varepsilon \ge 0)$ defined for $\varepsilon > 0$ by the Cauchy problem: $\varepsilon y'_t + y = 0$, $y(0, \varepsilon) = 0$; that is, $y = \exp(-t\varepsilon^{-1})$. The half-plane $\operatorname{Re}\varepsilon > 0$ has to be mapped by $\varepsilon = (1-u)/(1+u)$ into the disc $|u| < 1$.

{Reviewer's remarks: A slight error appears on p. 79: The derivative of $\ln(1+\theta x)$ is not $(1+\theta x)^{-1}$ but $\theta(1+\theta x)^{-1}$; therefore $f(i,k)$ does not converge to 0 but to $-\frac{1}{2}$. Lemma 1.3.4 results from a classical theorem: On a compact set a decreasing sequence of numerical continuous functions uniformly converges when it converges to 0 at each point. But this lemma is not proved by p. 81, which would have proved this theorem without the essential assumption that the sequence is decreasing.}

Reviewed by *Jean Leray*