MR639224 (82m:35020) 35E15 (35A99 35L30)
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Partial hyperbolicity and partial Gevrey classes.
J. Differential Equations **42** (1981), *no. 3*, 283–324.

Partial hyperbolicity was introduced by the reviewer [C. R. Acad. Sci. Paris Sér. A-B **276** (1973), A1685-A1687; MR0326138 (48 #4483)] and Y. Hamada, the reviewer and C. Wagschal [J. Math. Pures Appl. (9) **55** (1976), no. 3, 297–352; MR0435614 (55 #8572)] as a sufficient condition on operators with analytic coefficients for the Cauchy problem to be well-posed in classes of functions that belong to nonanalytic Gevrey classes with respect to some of the variables and are analytic in the other variables. This is the author's starting point; on the one hand he considers only operators with constant coefficients, but on the other hand he extends the classical results of Gårding and Hörmander and adapts their proofs to new Fréchet spaces $\gamma(d)$, to still larger spaces and to their duals.

He defines $\gamma(d)$ as follows:

$$x=(x_0,x',x'',x''')\in\mathbf{R}\times\mathbf{R}^q\times\mathbf{R}^{p-q}\times\mathbf{R}^{n-p}=\mathbf{R}^{n+1};$$

$f\in C^\infty(\mathbf{R}^{n+1})$; $\alpha=(\alpha_0,\alpha',\alpha'',\alpha''')=(\alpha_0,\cdots,\alpha_n)$ is a multi-index; let $d=(d_0,d',d'',d''')=(d_0,\cdots,d_n)$ with

$$0<d_1\le d_2\le\cdots\le d_q\le 1<d_{q+1}\le\cdots\le d_p<\infty=d_{p+1}=\cdots=d_n,$$

$d_0=d_n$; he writes $\alpha d=\sum_{j=0}^n\alpha_jd_j$ and $\overline{d}_j=d_j$ if $d_j<\infty$, $\overline{d}_j=0$ if $d_j=\infty$; the partial Gevrey class $\gamma(d)$ is the space of functions f such that, for each $l>0$, each compact $K\subset\mathbf{R}^{n+1}$ and each $N\ge 0$, there is some $C>0$ such that $|D^\alpha f|\le C(l\alpha\overline{d})^{\alpha\overline{d}}$ for $x\in K$ and all α satisfying $\alpha_0+|\alpha'''|\le N$; if $p=n$, no N enters. Hence f is an entire function of x', a function of x'' belonging to a Gevrey class and a C^∞ function of x''' and x_0.

The topic is the Cauchy problem $P(D)u=f$, $u-v=O(x_0^2)$, where $P(D)=\sum_\alpha a_\alpha D^\alpha=\sum_{j=0}^s Q_j(D',D'',D''')D_0^j$, $Q_s\ne 0$. This problem is said to be well posed in $\gamma(d)$ if it has a unique solution $u\in\gamma(d)$ for each choice of f and $v\in\gamma(d)$.

Among several theorems the main one asserts the Cauchy problem to be well posed in $\gamma(d)$ if and only if: Q_s is constant; $r=s$; $\alpha d^*\le sd_0^*$ if $a_\alpha\ne 0$, where $d_j^*=d_j$ when $d_j\le 1$ and $d_j^*=1$ when $d_j\ge 1$; $P(\tau N+i(0,\zeta',\eta'',\eta'''))\ne 0$ when $N=(1,0,\cdots,0)$, $(\tau,\zeta',\eta'',\eta''')\in\mathbf{C}\times\mathbf{C}^q\times\mathbf{R}^{p-q}\times\mathbf{R}^{n-p}$ and $|\mathrm{Re}\,\tau|>C(1+\sum_{j=1}^q|\zeta_j|^{1/d_j}+\sum_{j=q+1}^p|\eta_j|^{1/d_j})$ for some constant C.

Reviewed by *Jean Leray*