MR723603 (86c:35170) 35R35 (47H15 76B10)
Tôn, Bùi An (3-BC)
Free boundary problem for a two-layer inviscid incompressible fluid.
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The purpose of this paper is to prove the existence of a local (in time) classical solution. The approach to Lagrangian coordinates of Solonnikov is used; the method of successive approximations is applied twice, once correctly (p. 486). But the second application (p. 489, lines 15–16) reproduces a tempting error that the author committed on p. 178 of his earlier paper [J. Math. Anal. Appl. **94** (1983), no. 1, 166–180; MR0701455 (85i:35122)]. Let us point it out: Consider a Banach space X, a continuous map $F\colon X\to X$ and a bounded sequence $\{x_n\}$ of points of X such that $x_{n+1}=F(x_n)$; by Cantor's diagonalization extract a subsequence $\{x_{n_p}\}$ that weakly converges to a point x of X; the author's assertion that $x=F(x)$ is not well founded; even when the weak convergence of $\{x_{n_p}\}$ implies the weak convergence of $\{F(X_{n_p})\}$, i.e. of $\{x_{n_p+1}\}$ to some $x'=F(x)\in X$, then x' may differ from x. For instance, assume $F(x)\equiv -x$ and $x_1\neq 0$; then $x_n=(-1)^{n-1}x_1$; subsequences of $\{x_n\}$ can only converge to x_1 or $-x_1$, which do not satisfy $x=F(x)$.

Reviewed by *Jean Leray*