MR817771 (87c:35110) 35L70 (47H15 58E05)
Capozzi, A. (I-BARI); **Salvatore, A.** (I-SISSA)
On the equation $Lu = \nabla V(u)$. (Italian. English summary)
Differential problems and the theory of critical points (*Bari*, 1984), 41–63, *Coll. Atti Congr.*, *Pitagora*, *Bologna*, 1984.

The topic is the following problem. Let E be a real Hilbert space. Let $L\colon E \to E$ be a continuous selfadjoint operator, for which 0 is an isolated eigenvalue of finite multiplicity. Let $A\colon E \to E$ be a compact operator such that $A(0) = 0$, $A = \nabla\psi$, where $\psi \in C^1(E, \mathbf{R})$ and $\psi(0) = 0$. Find $u \in E$ such that $Lu = A(u)$ and $u \neq 0$. In other words: Find the nontrivial critical points of the functional $f \in C^1(E, \mathbf{R})$ defined by $f(u) = \frac{1}{2}(Lu, u)_E - \psi(u)$.

A useful review of printed or preprinted papers is made. Special attention is given to two cases satisfying neither the Palais-Smale nor the Cherami condition: the case of "strong resonance"; the case of "bounded potential", for which an unpublished approach is described. It is detailed when the problem is the following nonlinear wave problem: $u_{tt} - u_{xx} = f(x, t, u)$ and $u(x, t) = u(x, t+T)$ for all $t \in \mathbf{R}$ and $x \in [0, \pi]$, $u(0, t) = u(\pi, t) = 0$ for all $t \in \mathbf{R}$, T/π is rational, f is T-periodic in t, $f \in C^1([0, \pi] \times \mathbf{R} \times \mathbf{R}, \mathbf{R})$. Under some other very specific assumptions the existence of solutions is established.

Reviewed by *Jean Leray*