MR891945 (88e:35048) 35J10 (35Q20 81C05)
Yajima, Kenji
Existence of solutions for Schrödinger evolution equations.
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The Schrödinger initial value problem is: $i\partial u/\partial t = -\frac{1}{2}\Delta u + V(t,x)u$, $u(0) = u_0$, $t \in [-T,T]$, $x \in \mathbf{R}^n$. Define the Banach space $L^{q,\gamma}(T) = \{W\colon \int_{-T}^{T}[\int_{\mathbf{R}^n}|W(t,x)|^q\,dx]^{\gamma/q}\,dt < \infty\}$; and assume $V \in L^{p,\alpha}(T) + L^{\infty,\beta}(T)$, $p \geq 1$, $\alpha \geq 1$, $\beta > 1$, $0 \leq 1/\alpha < 1 - n/2p$, which allows moving singularities of type $|x|^{-2+\varepsilon}$ for $n \geq 4$ and $|x|^{-n/2+\varepsilon}$ for $n \leq 3$, $\varepsilon > 0$; then the problem has a unique regular solution for $u_0 \in L^2(\mathbf{R}^n)$; $\|u(t)\| = \|u_0\|$ for $|t| \leq T$; the problem generates a strongly continuous unitary propagator. More assumptions give more regularity.

Direct applications of the existing abstract theories lead to rather strong smoothness conditions. Here the characteristic features of Schrödinger equations are taken into account; suitable Banach spaces and estimates are introduced. The results follow from a contraction and from semigroup theory.

Reviewed by *Jean Leray*