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Régularité par rapport au temps des solutions faibles de l'équation de Navier-Stokes. **(French) [Regularity with respect to time for weak solutions of the Navier-Stokes equations]**

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Let Ω be a domain of $\mathbf{R}^3$ whose boundary Γ is compact and of class C^3. Define $V = \{v \in H_0^1(\Omega)\colon \operatorname{div} v = 0\}$. Assume that: $u \in L^2(0,T;V) \cap L^\infty(0,T;H)$ and $u \in L^s(0,T;L^r(\Omega))$ for some (r,s) such that $3 < r \leq \infty$, $2 < s \leq \infty$, $3/r + 2/s < 1$; either $f \in L^\alpha(0,T;H^{-\beta}(\Omega))$ for some (α,β) such that $0 \leq \beta < 1$, $\alpha > 2/(1-\beta)$, or $f \in C^\gamma([0,T];H^{-1}(\Omega))$ for some γ such that $0 < \gamma \leq 1$. Then $u\colon (0,T] \to V$ is strongly continuous: $u \in C^0((0,T];V)$.

The increase in the regularity of f increases the regularity of u. In particular: Assume also $\mathcal{O}$ to be an open neighborhood of $(0,T)$ in $\mathbf{C}$ and f to have an analytic continuation $\mathcal{O} \to H^{-1}(\Omega)$; then u has an analytic continuation $\mathcal{O}' \to V$ to some neighborhood $\mathcal{O}'$ of $(0,T)$ in $\mathcal{O}$.

Finally the following assumption is added: $f\colon \Omega_0 \times \mathcal{O} \to \mathbf{C}^3$ is holomorphic where Ω_0 is some open subset of Ω.

The proof uses a semigroup generated by a selfadjoint, strictly positive definite operator.

Reviewed by *Jean Leray*