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Borchers, Wolfgang (D-PDRB); **Sohr, Hermann** (D-PDRB)

On the semigroup of the Stokes operator for exterior domains in L^q-spaces.

Math. Z. **196** (1987), *no. 3,* 415–425.

Let Ω be an exterior domain in $\mathbf{R}^n (n \geq 3)$, with compact boundary of class $C^{2+\mu}$ $(0 < \mu < 1)$. On Ω, the Stokes boundary value problem $-\Delta u + \nabla p = f$, $\operatorname{div} u = 0$, $u|_{\partial\Omega} = 0$, is studied for $u \in L^q(\Omega)^n$, $1 < q < \infty$, $p \in L^q_{\rm loc}(\Omega)$. Let P_q be the projection operator of $L^q(\Omega)^n$ onto the subspace $H_q(\Omega)$ of divergence-free fields with zero normal component on $\partial\Omega$; the Stokes operator is $A_q = -P_q\Delta$. It is known that $-A_q$ generates an analytic semigroup $\exp(-tA_q)$ $(t \geq 0)$ in $H_q(\Omega)$. The following new properties are proved: Let $0 < \omega < \pi/2$; for all $\lambda \in \mathbf{C}$ with $|\lambda| > 0$ and $|\arg\lambda| < \omega + \pi/2$, the resolvent $(\lambda I + A_q)^{-1}$ exists and satisfies $|\lambda| \cdot \|(\lambda I + A_q)^{-1}\|_{H_q(\Omega)} \leq c(q,\omega,\Omega)$; $t \mapsto \exp(-tA_q)$ is well defined and analytic for $t \in \mathbf{C}$, $t \neq 0$, and $|\arg t| < \omega$. If $0 < \varepsilon < \omega$, $t \in \mathbf{C}$, $t \neq 0$, $|\arg t| < \omega - \varepsilon$ and $k = 0, 1, 2, \cdots$, then $|t|^k \cdot \|A_q^k \exp(-tA_q)\|_{H_q(\Omega)} < C(q,\omega,\varepsilon,\Omega)$. Moreover $\lim_{t\to\infty} |\exp(-tA_q) \cdot v\|_{H_q(\Omega)} = 0$ for any $v \in H_q(\Omega)$.

Reviewed by *Jean Leray*

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