MR905619 (88k:35164) 35Q10 (35B35 35D99)
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L^2 stability for weak solutions of the Navier-Stokes equations in $\mathbf{R}^3$.
Indiana Univ. Math. J. **36** (1987), *no. 3,* 685–691.

The following theorem is proved. Let $L^2(\mathbf{R}^3)$ be the space of the square integrable functions $\mathbf{R}^3 \to \mathbf{R}^3$ and $\|\cdot\|$ be its norm. Let $H = \{\varphi \in L^2(\mathbf{R}^3)\colon \operatorname{div}\varphi = 0\}$. Let $H^p = H^p(\mathbf{R}^3)$ be the Sobolev space of order p. Consider the initial value problem for the nonstationary Navier-Stokes equations in the whole space $\mathbf{R}^3$ with a given external force $\in L^1(0,\infty;L^2)\cap L^2(0,\infty;L^2)$. Assume that for some initial velocity $v_0 \in H$ there exists a strong solution $v \in H^1(0,\infty;L^2)\cap L^2(0,\infty;H^2)$. Then for any initial velocity $u_0 \in H$ there exists a suitable weak solution $u \in L^\infty(0,\infty;H)\cap L^2(0,\infty;H^1)$ such that $\|u(t)-v(t)\| \to 0$ as $t\to\infty$. For the construction of suitable weak solutions see a paper by H. Beirão da Veiga [J. Math. Pures Appl. (9) **64** (1985), no. 3, 321–334; MR0823407 (87h:35268)].

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