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Hsu, Pei [Hsu, Elton P.] (1-NY-X)
On the Poisson kernel for the Neumann problem of Schrödinger operators.
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Summary: "Let D be a bounded domain in $\mathbf{R}^d$ $(d \geq 3)$ and let $b(t,x,y)$ be the kernel of the Feynman-Kac semigroup associated with the reflecting Brownian motion $\{X_t\colon t \geq 0\}$ and potential V, namely

$$E^x\big[\exp\big(\int_0^t V(X_s)\,ds\big)\,f(X_t)\big] = \int_D b(t,x,y)f(y)m(dy).$$

We assume that V is in the Kato class K_d [see M. Aizenman and B. Simon, Comm. Pure Appl. Math. **35** (1982), no. 2, 209–273; MR0644024 (84a:35062)]. The Poisson kernel studied in this paper is $N_V(x,y) = \int_0^\infty b(t,x,y)\,dt$. In general N_V may be infinite. We show that if $N_V(x,y)$ is finite for one pair of points then it is finite for all $x \neq y$ and there exist two constants c_1, c_2 (depending on D and V) such that $c_1 \leq \|x-y\|^{d-2} N_V(x,y) \leq c_2$. This happens precisely when the spectrum of $H_V = \Delta/2 + V$ under the Neumann boundary condition lies in the negative half-axis. This result is used to discuss the Neumann boundary value problem of H_V. We prove that for any boundary function $f \in L^\alpha(\partial D)$, $\alpha \geq 1$, the problem has a unique weak solution $u_f(x) = \frac{1}{2}\int_{\partial D} N_V(x,y)f(y)\sigma(dy) \in C(D)$ and its growth rate near the boundary can be estimated by $\|f\|_{\alpha,\partial D}$."

Reviewed by *Jean Leray*