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On L^2 well posedness of the Cauchy problem for Schrödinger type equations on the Riemannian manifold and the Maslov theory.

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Let M be a C^∞ complete Riemann manifold with a countable basis and without boundary. Denote by L^2 the Hilbert space of square-integrable functions on M. In the distribution sense consider the Cauchy problem for the "nonlinear Schrödinger equation" $[-i\partial_t - \frac{1}{2}\Delta + \mathbf{B} + C]u = f$, where Δ denotes the Laplace-Beltrami operator on M, C a given C^∞ function on M, $\mathbf{B}$ a given complexified C^∞ vector field on M and $\mathbf{B}u$ the Lie derivative; the unknown function u and the given one f are continuous functions of $t \in [0,T]$ or $t \in [-T,0]$ $(T > 0)$ with values in L^2; the value of u at $t = 0$ is given. Denote by $\omega_\mathbf{B}$ the 1-form on M defined by $\mathbf{B}$. The following theorem is proved: If the Cauchy problem is well posed, then the following condition holds: $\sup_{\gamma\in\Gamma} |\int_\gamma \operatorname{Re}\omega_\mathbf{B}| < \infty$, where Γ is the family of all geodesics on M.

The proof deduces from the well-posedness of the problem an estimate of its solutions. Then, assuming the preceding condition not satisfied and using Maslov's construction of asymptotic solutions of the homogeneous problem (i.e. $f = 0$), the proof constructs nonhomogeneous problems with solutions contradicting that estimate.

Reviewed by *Jean Leray*