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On energy inequality, smoothness and large time behavior in L^2 for weak solutions of the Navier-Stokes equations in exterior domains.

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Let Ω be an exterior domain in $\mathbf{R}^n$ $(n=3,4)$ with smooth $\partial\Omega$. On $\Omega\times[0,\infty)$ consider the Navier-Stokes problem: $u_t-\Delta u+u\cdot\nabla u+\nabla p=f$ and $\nabla u=0$ in $\Omega\times(0,\infty)$, $u=0$ on $\partial\Omega\times(0,\infty)$, $u(x,0)=a(x)$, with u and p unknown, a and f given. Introduce the energy inequality

$$\|u(t)\|_2^2+2\int_{(s,t)}\|\nabla u\|_2^2 d\tau\le\|u(s)\|_2^2+2\int_{(s,t)}(f,u)d\tau$$

for $s=0$, a.e. $s>0$, and all $t\ge s$. Denote by X_2 the L^2-closure of the set of the smooth solenoidals vector fields with compact support in Ω. Theorem 1: Let $n=3$ or 4 and suppose that $f\in L^2(0,T;X_2)$ for all $T>0$; then there is a weak solution of the Navier-Stokes problem satisfying the energy inequality. If in addition $f\in L^1(0,\infty;X_2)$, then any weak solution satisfying that inequality is such that $\lim_{t\to\infty}\|u(t)\|_2=0$; if moreover f is smooth enough and $f\in L^2(0,\infty;X_2)$, then there is a $T_0>0$ depending only on a, f, Ω such that u is a classical solution in the interval (T_0,∞).

Assume now $n=3$ and w to be a stationary solution of Finn's type; Theorem 2 similarly gives solutions u such that $\lim_{t\to\infty}\|u(t)-w(t)\|_2=0$.

Reviewed by *Jean Leray*