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Deuring, P.; **von Wahl, W.**; **Weidemaier, P.**

Das lineare Stokes-System in R^3. I. Vorlesungen über das Innenraumproblem. (German)
[The linear Stokes system in R^3. I. Lectures on the interior problem]

Bayreuth. Math. Schr. No. 27 (1988), vi+252 *pp*.

This is the first of a series of monographs publishing von Wahl's lectures, whose motivation is his opinion that O. A. Ladyzhenskaya has only suggested the proofs of her assertions in her well-known work *The mathematical theory of viscous incompressible flow* [Gordon & Breach, New York, 1963; MR0155093 (27 #5034b); second edition, 1969; MR0254401 (40 #7610)]. In his excellent review of that second edition, R. Finn writes: "This volume exhibits the spirit of modern applied mathematics in one of its best and most fruitful forms"; he also makes many accurate critical remarks.

The general topic of these monographs is Navier-Stokes equations. The first monograph under review here deals with only the time independent linear Stokes equations: $-\nu\Delta u_i+\partial\pi/\partial x_i=f_i$, $\sum_i\partial u_i/\partial x_i=0$ in A, where A is a domain of $\mathbf{R}^3$ such that ∂A is smooth and either A or $\mathbf{R}^3\setminus A$ is bounded; $\nu=\mathrm{const}$; $i=1,2,3$; the functions f_i are given on A. The problem to be solved is the following: Determine u_i and π on $\overline{A}$ when, on ∂A, the functions u_i are given, such that $\iint_{\partial A}u_1dx_2\wedge dx_3+u_2dx_3\wedge dx_1+u_3dx_1\wedge dx_2=0$. The following is carefully proved: under suitable assumptions this problem has a unique solution, differentiable in A, such that the u_i are continuous on $\overline{A}$; if A is bounded, then, under other assumptions, the second derivatives of u_i and the first derivatives of π belong to $L_p(A)$. Ladyzhenskaya's integral equation method is applied; it makes use of Oseen and Odqvist potentials, but those two names are not quoted. Many other mathematical techniques are used and clearly detailed or quoted. This monograph is said to be written for students—I would say for very studious students, able to master some 58 lemmata and 18 theorems.

Reviewed by *Jean Leray*