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On approximate inertial manifolds to the Navier-Stokes equations.
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Section 1 of this paper reviews the theory of the 2-dimensional Navier-Stokes equations with initial data and either the homogeneous Dirichlet boundary condition or the double periodicity condition. Paragraph 2.1 recalls the concepts of inertial manifold (IM) and approximate IM. Theorem 2.1 states results of C. Foias, O. Manley and R. Temam [C. R. Acad. Sci. Paris Sér. I Math. **305** (1987), no. 11, 497–500; MR0916319 (88j:76026); RAIRO Modél. Math. Anal. Numér. **22** (1988), no. 1, 93–118; MR0934703 (89h:76022)]; to indicate their importance an illustrative example is presented. Paragraph 2.2 recalls the analytic manifold M^s due to Foias, J.-C. Saut and Temam [Foias and Temam, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) **5** (1978), no. 1, 28–63; MR0481645 (58 #1749); Foias and Saut, ibid. Cl. Sci. (4) **10** (1983), no. 1, 169–177; MR0713114 (85h:35171)]. It is an approximate IM, which is a C-analytic manifold; the orbits in the universal (global) attractor are closer to M^s than to the finite-dimensional manifold M_0 introduced by Foias, Manley and Temam; M_s leads to sharp inequalities. But M_0 is explicitly known, whereas M_s is constructed by the contraction principle. For that reason paragraph 2.3 gives explicit approximations of M^s that approximate the universal attractor as well as M^s does. Those results can be used for real computations. Several of them can be extended to the 3-dimensional Navier-Stokes equations.

The error estimates are similar to those obtained in Temam's recent asymptotic approximations [C. R. Acad. Sci. Paris Sér. II Méc. Phys. Chim. Sci. Univers Sci. Terre **306** (1988), no. 6, 399–402; MR0979153 (90a:76067)].

Reviewed by *Jean Leray*