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Practical methods for the uniform asymptotic evaluation of oscillating integrals with several coalescing saddle points.

Asymptotic and computational analysis (*Winnipeg, MB*, 1989), 137–173, *Lecture Notes in Pure and Appl. Math.*, 124, *Dekker, New York*, 1990.

The integrals considered are $I(\alpha)=\int_{-\infty}^{\infty} g(t)\exp[if(\alpha;t)/\hbar]dt$, $\hbar\to 0$, where f and g are analytic functions; $f(\alpha;t)$ is real and $\alpha\in\mathbf{R}^p$. The behaviour of $I(\alpha)$ does not change if $I(\alpha)$ is computed on a neighbourhood of the saddle points of the function $t\mapsto f(\alpha;t)$. Those saddle points depend on α and can coalesce as α varies. The behaviour of $I(\alpha)$ can be expressed in terms of "cuspoid" canonical integrals and of their derivatives. For n coalescing saddle points the "cuspoid" canonical integral is

$$C_{n+1}(a)=\int_{-\infty}^{\infty}\exp[i(u^{n+1}+a_1u+\cdots+a_{n-1}u^{n-1})]du.$$

For $n=2$ the result is easy; $C_3(a_1)$ is Airy's function. The purpose of the paper is to discuss the two practical techniques that have been applied and then to outline their results in the cases $n=3$ and $n=4$. The first technique performs the numerical quadrature along a convenient path in the complex u plane. The second technique computes $C_{n+1}(a)$ by numerically integrating a set of differential equations satisfied by $C_{n+1}(a)$. Some of the results are shown by a contour plot and thirteen perspective plots. There are 70 references.

Reviewed by *Jean Leray*