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Asymptotic approach to singular solutions for nonlinear elliptic equations involving critical Sobolev exponent. (French summary)
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The author establishes results related to a Brézis-Peletier conjecture about the blowing up of solutions [H. Brézis and L. A. Peletier, in *Partial differential equations and the calculus of variations, Vol. I*, 149–192, Birkhäuser Boston, Boston, MA, 1989; MR1034005 (91a:35030)]. Let Ω be a smooth bounded domain in $\mathbf{R}^N$ with $N \geq 3$. Denote by $p=(N+2)/(N-2)$ the critical exponent; let u_ε be a subcritical solution; i.e. let $\varepsilon>0$, $-\Delta u_\varepsilon = N(N-2)u_\varepsilon^{p-\varepsilon}$ in Ω, $u_\varepsilon>0$ in Ω, $u_\varepsilon=0$ on $\partial\Omega$. The existence of u_ε is well known. Assume that $\lim_{\varepsilon\to 0}\int_\Omega |\nabla u_\varepsilon|^2/\|u_\varepsilon\|^2_{L^{p+1-\varepsilon}(\Omega)} = S_N$, where $S_N=\pi N(N-2)[\Gamma(N/2)/\Gamma(N)]$ is the best Sobolev constant in $\mathbf{R}^N$. After passing to a subsequence, when $\varepsilon\to 0$, there exists $x_0\in\Omega$ such that $\lim u_\varepsilon = 0$ in $C^1(\Omega\setminus\{x_0\})$ and

$$\lim |\nabla u_\varepsilon|^2 = N(N-2)[S_N/N(N-2)]^{N/2}\delta_{x_0}$$

in the sense of distributions; δ is the Dirac distribution; x_0 is a critical point of an explicitly given function; $\lim \varepsilon\|u_\varepsilon\|^2_{L^\infty(\Omega)}$ and $\lim \varepsilon^{-1/2}u_\varepsilon$ are also explicitly given functions.

A similar result is established when u_ε is a solution of the problem $-\Delta u_\varepsilon = N(N-2)u_\varepsilon^p + \varepsilon u_\varepsilon$ in Ω, $u_\varepsilon>0$ in Ω, $u_\varepsilon=0$ on $\partial\Omega$.

Reviewed by *Jean Leray*