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Continuity of solutions of Schrödinger equations.
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Let m be the Lebesgue measure on $\mathbf{R}^d$, where $d \geq 2$. Let μ be a signed Radon measure on an open subset Ω of $\mathbf{R}^d$; let $|\mu|$ be its total variation measure; let $|\mu|_s = |\mu| - (d|\mu|/dm)m$ be its m-singular part. Let $N(x,y)$ be the Newtonian kernel on $\mathbf{R}^d$, that is, $-\log|x-y|$ for $d = 2$ and $|x-y|^{2-d}$ for $d \geq 3$. The measure μ is said to be of Kato class if, for every y in Ω: $\lim_{r\downarrow 0}(\sup_{|x-y|<r}\int_{|\xi-y|<r} N(x,\xi)\,d|\mu|(\xi)) = 0$.

The time-independent Schrödinger equation under consideration is $(-\Delta+\mu)u = 0$. If the function u belongs to $L^1_{\rm loc}(\Omega, |\mu|+m)$ and satisfies $-\int_\Omega u(\xi)\Delta\varphi(\xi)\,dm(\xi) + \int_\Omega u(\xi)\varphi(\xi)\,d\mu(\xi) = 0$ for every test function φ in $C^\infty_0(\Omega)$, then u is said to be a distributional solution on Ω of the above Schrödinger equation. Assume there are a subset X of Ω and a function $\tilde{u}$ in $L^1_{\rm loc}(\Omega, |\mu| + m)$, continuous at each point of X as a function on Ω, and such that $u = \tilde{u}$, $(|\mu|+m)$-a.e. on Ω; then u is said to be continuous on X. This paper gives an elementary proof to the following theorem: A distributional solution u on Ω is continuous on Ω if and only if u is continuous on Ω except for a subset of Ω of $|\mu|_s$-measure zero. Corollary 1: A distributional solution u on Ω is continuous on Ω if and only if u is continuous on $\operatorname{supp}|\mu|_s$. Corollary 2 (Aizenmann and Simon, 1982): If μ is of Kato class and is m-absolutely continuous, then every distributional solution is continuous. Corollary 3: Any distributional solution on Ω which is continuous on Ω except for a discrete subset of Ω is continuous on the whole of Ω.

Reviewed by *Jean Leray*