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dans les *Mathematical reviews*

82c:81039

Bros, Jacques; Pesenti, Danielle

Fredholm theory in complex manifolds with complex parameters: analyticity properties and Landau singularities of the resolvent.

J. Math. Pures Appl. (9) 59 (1980), no. 3, 375–401.

Let Δ be a domain in C^q, M a complex n-dimensional manifold and S a closed subset of $\Delta \times M$ such that S and $D = \Delta \times M \setminus S$ are trivial fiber spaces with base Δ; let S_k and D_k be their fibers above $k \in \Delta$; let $\hat{\Delta}$ be the universal covering space of Δ; denote by $\hat{k}$ any point of $\hat{\Delta}$ projecting onto k; let $h(\hat{k}) \in H_n(D_k)$ be a compact n-dimensional homology class of D_k depending continuously on $\hat{k}$; let $\omega(k,z)$ be a holomorphic $(0,n)$-differential form in $\Delta \times M$ and let $G(k,z,z')$ be an analytic function of $(k,z) \in D$ and $z' \in M$; let $F(\hat{k},z,z',\lambda)$ be the Fredholm resolvent of the kernel G, that is, the solution of $F(\hat{k},z,z',\lambda) = G(k,z,z') + \lambda \int_{h(\hat{k})} F(\hat{k},z,z_1,\lambda)G(k,z_1,z')\omega(k,z)$, Fredholm's fixed integration domain being thus replaced by $h(\hat{k})$.

Fredholm's argument applies; therefore F is a meromorphic function of the parameter $\lambda \in \mathbb{C}$; this function is holomorphic in $\hat{k} \in \hat{\Delta}$, $z \in M \setminus S_k$ and $z' \in M$.

From now on assume $S = S^1 \cup \cdots \cup S^m$, $m < n-1$, S^i being regular complex analytic hypersurfaces of $\Delta \times M$. The simple critical points a of the stratum $A = S^1 \cap \cdots \cap S^m$ correspond in the fiber to simple or quadratic pinches of the manifolds $S^1, \cdots, S^m$; these points a project onto the Landau variety LA, which is a regular complex analytic hypersurface of Δ; call its local equation $l(k) = 0$. Given a, its projection k_a in LA and some $\hat{k}_a \in \hat{\Delta}$ with projection k_a; then $F(\hat{k},z,z',\lambda)$, in the neighborhood of $\hat{k}_a$, if considered as a function of k, admits a branching singularity on LA. It is studied under the following assumption: G has a first order polar singularity on each S^i.

If $n-m$ is even, then in a neighborhood of $\hat{k}_a$, $F(\hat{k},z,z',\lambda) = F_1(k,z,z',\lambda) + l(k)^{(n-m-1)/2}F_2(k,z,z',\lambda)$, where the $F_i (i = 1,2)$ are meromorphic; thus one has to use in that neighborhood the two-sheeted covering of Δ ramifying on LA.

If $n-m$ is odd, then a less simple result is given; in a neighborhood of $\hat{k}_a$, one has to use the infinite-sheeted covering of Δ ramifying on LA.

Arguments are based on the Picard-Lefschetz formula about vanishing cycles.

The introduction shows how interesting these results and their possible extensions are for quantum field theory.

82d:34061

Gingold, H.

A new basis for singularly perturbed problems: a representation theorem.

Adv. in Appl. Math. 1 (1980), no. 1, 67–107.

The author provides the following contribution to singular perturbation theory: Let $y(t,u) = \sum_\nu y_\nu(t)u^\nu$ be a holomorphic function of $u (u \in \mathbb{C}, |u| < 1)$ continuously depending on $t (0 \leq t \leq 1)$; assume $y_\nu(t) = O(\nu^\alpha)$ with $\alpha < \frac{1}{2}$; assume $y(t, \cos\theta)$ to be an integrable function of θ on

then an expansion of $y(t,u)$ on $-1 < u \leq 1$ is given in terms of Čebyšev polynomials; its coefficients are continuous functions of t; using the CesÃ ro summation process it converges even for $u = 1$.

The case $0 \leq t < \infty$ and the behaviour for $t \to \infty$ are also studied. This applies for example to the continuous function $y(t,\varepsilon)$ of $t (0 \leq t < \infty)$ and $\varepsilon (\varepsilon \geq 0)$ defined for $\varepsilon > 0$ by the Cauchy problem: $\varepsilon y'_t + y = 0$, $y(0,\varepsilon) = 0$; that is, $y = \exp(-t\varepsilon^{-1})$. The half-plane $\operatorname{Re}\varepsilon > 0$ has to be mapped by $\varepsilon = (1-u)/(1+u)$ into the disc $|u| < 1$.

{Reviewer's remarks: A slight error appears on p. 79: The derivative of $\ln(1+\theta x)$ is not $(1+\theta x)^{-1}$ but $\theta(1+\theta x)^{-1}$; therefore $f(i,k)$ does not converge to 0 but to $-\frac{1}{2}$. Lemma 1.3.4 results from a classical theorem: On a compact set a decreasing sequence of numerical continuous functions uniformly converges when it converges to 0 at each point. But this lemma is not proved by p. 81, which would have proved this theorem without the essential assumption that the sequence is decreasing.

82j:35028

Yoshino, Masafumi

On the solvability of Goursat problems and a function of number theory.

Proc. Japan Acad. Ser. A Math. Sci. 57 (1981), no. 6, 294–296.

The following reduced Goursat problem with constant coefficients is studied:

$$(a\partial_1^{-1}\partial_2+\varepsilon+b\partial_1\partial_2^{-1}+c\partial_1^2\partial_2^{-2})u=h(x)$$

where $x=(x_1,x_2)\in\mathbb{C}^2$, $\partial_i=\partial/\partial x_i(i=1,2)$ and ∂_i^{-1} denotes integration with respect to the variable x_i from the origin to x_i. The characteristic roots are the roots of the equation $a\lambda^3+\varepsilon\lambda^2+b\lambda+c=0$. Solvability and uniqueness theorems are stated, using several nontrivial arithmetical properties of the characteristic roots; one of them introduces a new function of number theory. Those theorems supplement preceding results by the reviewer [J. Math. Pures Appl. (9) 53 (1974), 133–136; MR 0367411 (51 #3653)], the reviewer and C. Pisot [ibid. (9) 53 (1974), 137–145; MR 0366856 (51 #3102)], and S. Alinhac [Comm. Partial Differential Equations 1 (1976), no. 3, 231–282; MR 0415082 (54 #3173)].

82j:35029
Yoshino, Masafumi
On the solvability of Goursat problems and a function of number theory.
Duke Math. J. 48 (1981), no. 3, 685–696.
The author proves what he asserts in the paper reviewed above [MR0628113 (82j:35028)]. He shows the sharpness of another result of his about the nonlinear Goursat problem in two variables. And he studies the linear Goursat problem of Hermitian type with constant coefficients: $\varepsilon u(x)+\sum_\alpha a_\alpha D^\alpha u(x)=h(x)$, where ε and $a_\alpha=\overline{a}_{-\alpha}\in\mathbb{C}$, $x\in\mathbb{C}^d$, $\alpha=(\alpha_1,\cdots,\alpha_d)\in A\subset Z^d$, A is finite, $\alpha_1+\cdots+\alpha_d=0$ on A; h is given; u is unknown; h and u are analytic at the origin. He gives an expansion in polynomial eigenfunctions that is defined and is the unique solution if $\varepsilon\notin E$, where $E\subset\mathbb{R}$, meas$(E)=0$; E is bounded, E is the set of zeros of a function ρ defined as follows by means of a bounded sequence $N\ni k\mapsto\varepsilon_k\in\mathbb{R}$ and of a sequence $N\ni k\mapsto n(k)\in N$ tending to infinity: $\rho(\varepsilon)=\liminf_{k\to\infty}|\varepsilon-\varepsilon_k|^{1/n(k)}$.

82k:35024
Yoshino, Masafumi
On the solvability of nonlinear Goursat problems.
Proc. Japan Acad. Ser. A Math. Sci. 57 (1981), no. 5, 247–248.
The nonlinear analytic Cauchy-Goursat problem $\varepsilon D^\beta u=a(x,D^\alpha u)$, $u=O(x^\beta)$, is studied in the neighborhood of the origin of C^d, where $|\alpha|\leq\beta$, $\alpha\neq\beta$ and ε is a complex constant. A theorem is stated which supplements preceding results by L. Gårding [Acta Math. 114 (1965), 143–158; MR 0176221 (31 #496)] and C. Wagschal [J. Math. Pures Appl. (9) 58 (1979), no. 3, 309–337; MR 0544256 (82m:35024)]. The assumptions of that theorem are very special when $d\geq 3$.

82m:35020
Persson, Jan
Partial hyperbolicity and partial Gevrey classes.
J. Differential Equations 42 (1981), no. 3, 283–324.
Partial hyperbolicity was introduced by the reviewer [C. R. Acad. Sci. Paris Sér. A-B 276 (1973), A1685-A1687; MR 0326138 (48 #4483)] and Y. Hamada, the reviewer and C. Wagschal [J. Math. Pures Appl. (9) 55 (1976), no. 3, 297–352; MR 0435614 (55 #8572)] as a sufficient condition on operators with analytic coefficients for the Cauchy problem to be well-posed in classes of functions that belong to nonanalytic Gevrey classes with respect to some of the variables and are analytic in the other variables. This is the author's starting point; on the one hand he considers only operators with constant coefficients, but on the other hand he extends the classical results of Gårding and Hörmander and adapts their proofs to new Fréchet spaces $\gamma(d)$, to still larger spaces and to their duals.

He defines $\gamma(d)$ as follows:

$$x=(x_0,x',x",x"')\in\mathbb{R}\times R^q\times R^{p-q}\times\mathbb{R}^{n-p}=\mathbb{R}^{n+1};$$

$f\in C^\infty(\mathbb{R}^{n+1})$; $\alpha=(\alpha_0,\alpha',\alpha",\alpha"')=(\alpha_0,\cdots,\alpha_n)$ is a multi-index; let $d=(d_0,d',d",d"')=(d_0,\cdots,d_n)$ with

$$0<d_1\leq d_2\leq\cdots\leq d_q\leq 1<d_{q+1}\leq\cdots\leq d_p<\infty=d_{p+1}=\cdots=d_n,$$

$d_0=d_n$; he writes $\alpha d=\sum_{j=0}^n\alpha_jd_j$ and $\overline{d}_j=d_j$ if $d_j<\infty$, $\overline{d}_j=0$ if $d_j=\infty$; the partial Gevrey class $\gamma(d)$ is the space of functions f such that, for each $l>0$, each compact $K\subset R^{n+1}$ and each $N\geq 0$, there is some $C>0$ such that $|D^\alpha f|\leq C(l\alpha\overline{d})^{\alpha\overline{d}}$ for $x\in K$ and all α satisfying $\alpha_0+|\alpha"'|\leq N$; if $p=n$, no N enters. Hence f is an entire function of x', a function of $x"$ belonging to a Gevrey class and a C^∞ function of $x"'$ and x_0.

The topic is the Cauchy problem $P(D)u=f$, $u-v=O(x_0^2)$, where $P(D)=\sum_\alpha a_\alpha D^\alpha=\sum_{j=0}^s Q_j(D',D",D"')D_0^j$, $Q_s\neq 0$. This problem is said to be well posed in $\gamma(d)$ if it has a unique solution $u\in\gamma(d)$ for each choice of f and $v\in\gamma(d)$.

Among several theorems the main one asserts the Cauchy problem to be well posed in $\gamma(d)$ if and only if: Q_s is constant; $r = s$; $\alpha d^* \leq s d_0^*$ if $a_\alpha \neq 0$, where $d_j^* = d_j$ when $d_j \leq 1$ and $d_j^* = 1$ when $d_j \geq 1$; $P(\tau N + i(0, \zeta', \eta", \eta"')) \neq 0$ when $N = (1, 0, \cdots, 0)$, $(\tau, \zeta', \eta", \eta"') \in \mathbb{C} \times \mathbb{C}^q \times \mathbb{R}^{p-q} \times R^{n-p}$ and $|\mathrm{Re}\,\tau| > C(1 + \sum_{j=1}^q |\zeta_j|^{1/d_j} + \sum_{j=q+1}^p |\eta_j|^{1/d_j})$ for some constant C.

83g:81018
Weinstein, Alan; Zelditch, Steven
Singularities of solutions of some Schrödinger equations on $\mathbb{R}^n$.
Bull. Amer. Math. Soc. (N.S.) 6 (1982), no. 3, 449–452.
This is an announcement of a publication improving on the second author's Ph.D. Thesis ["Reconstruction of singularities for solutions of Schrödinger equations", Ph.D. Thesis, Univ. California, Berkeley, Calif., 1981] and a paper by the first author ["A symbol class for some Schrödinger equations on $\mathbb{R}^n$", Amer. J. Math., to appear]. It deals with Schrödinger's equation

$$2i\hbar\partial\psi/\partial t = \Big[\sum_j(-\hbar^2\partial^2/\partial x_j^2 + \omega_j^2 x_j^2) + 2V(x)\Big]\psi;$$

for each $m \geq 0$ the derivatives of order m of V decrease at infinity at least as fast as $|x|^{-m}$. Such a V has influence neither on the WF of the solution to the Cauchy problem, nor on the hyperplanes containing the singular support of the fundamental solution (i.e., the propagator) when they exist, i.e., when, for some j, $\hbar\omega_j/\pi$ is an integer. It is proven by means of the "bi-symbol" of pseudodifferential operators.

83m:35118
Chazarain, J.
Sur le comportement semi-classique de l'amplitude de diffusion d'un hamiltonien quantique.
Goulaouic-Meyer-Schwartz Seminar, 1980–1981, Exp. No. V, 8 pp., École Polytech., Palaiseau, 1981.
Let $-h^2\Delta + V(x)$ be an operator in $\mathbb{R}^n$, where h is a small parameter; the potential V is C^∞ and zero outside the ball $B\colon |x| < R$, and is not assumed to be radial. The author uses semiclassical approximation to study the scattering of the plane wave $\exp(ih^{-1}kx \cdot \omega)$, where $\omega \in S^{n-1}$. Let P be the tangent plane at $-R\omega$ to the sphere $\partial B\colon |x| = R$, and let $x(t,y)$, $\xi(t,y)$ be the trajectory, defined by the Cauchy data $x(0,y) = y \in P$, $\xi(0,y) = k\omega$, of the field of the Hamiltonian $|\xi|^2 + V(x)$. For all $y \in P$, this trajectory is assumed to exit from B for sufficiently large t. For the Hamiltonian $|\xi|^2 + V(x) - k$, the radii leaving B are the elements of a Lagrangian manifold Λ_0. Theorem: The scattering amplitude in the direction $\theta \in S^{n-1}$ is an oscillating Maslov function of degree 0 with respect to the Lagrangian manifold Λ_0. The introduction of a caustic allows the author to make this result explicit.

85i:35122
Tôn, Bùi An
On the Euler equations for nonhomogeneous fluids.
J. Math. Anal. Appl. 94 (1983), no. 1, 166–180.
This paper intends to present an elementary proof, simpler than the previous ones, for the existence of a local regular solution of the Euler problem for a nonhomogeneous fluid in a fixed domain. But the sketched proof of Lemma 4.2 is not a "well-known argument"; it is a misuse of subsequences. From a sequence $\{v_n\}$ of points belonging to a compact set, one can extract a subsequence $\{v_{n_p}\}$ such that both v_{n_p} and v_{n_p-1} have limits; if one could always realize the identity of these two limits, then the proof of Lemma 4.2 would hold.

86c:35170
Tôn, Bùi An
Free boundary problem for a two-layer inviscid incompressible fluid.
Math. Methods Appl. Sci. 5 (1983), no. 4, 476–490.
The purpose of this paper is to prove the existence of a local (in time) classical solution. The approach to Lagrangian coordinates of Solonnikov is used; the method of successive approximations is applied twice, once correctly (p. 486). But the second application (p. 489, lines 15–16) reproduces a tempting error that the author committed on p. 178 of his earlier paper [J. Math. Anal. Appl. 94 (1983), no. 1, 166–180;MR 0701455 (85i:35122)]. Let us point it out: Consider a Banach space X, a continuous map $F\, X \to X$ and a bounded sequence $\{x_n\}$ of points of X such that $x_{n+1} = F(x_n)$; by Cantor's diagonalization extract a subsequence $\{x_{n_p}\}$ that weakly converges to a point x of X; the author's assertion that $x = F(x)$ is not well founded; even when the weak convergence of $\{x_{n_p}\}$ implies the weak convergence of $\{F(X_{n_p})\}$, i.e. of $\{x_{n_p+1}\}$ to some $x' = F(x) \in X$, then x' may differ from x. For instance, assume $F(x) \equiv -x$ and $x_1 \neq 0$; then $x_n = (-1)^{n-1}x_1$; subsequences of $\{x_n\}$ can only converge to x_1 or $-x_1$, which do not satisfy $x = F(x)$.

86e:35122

Okamoto, Hisashi
On the equation of nonstationary stratified fluid motion: uniqueness and existence of the solutions.
J. Fac. Sci. Univ. Tokyo Sect. IA Math. 30 (1984), no. 3, 615–643.
Let Ω be a bounded domain with smooth boundary in $\mathbb{R}^n$, where n is 2 or 3. The motion of a viscous inhomogeneous incompressible fluid with velocity u, pressure p, density ρ and viscosity 1 is governed by the system $\partial\rho/\partial t + u \cdot \nabla\rho = 0$, $\rho\{\partial u/\partial t + (u \cdot \nabla)u\} = \Delta u - \nabla p$, $\operatorname{div} u = 0$ $(0 < t;\ x \in \Omega)$ and the boundary conditions, where a and ρ_0 are data, $u|_{\partial\Omega} = 0$, $u|_{t=0} = a(x)$, $\rho|_{t=0} = \rho_0(x)$. O. A. Ladyzhenskayaand V. A. Solonnikov [J. Soviet Math. 9 (1978), 697–749; `MR 0425391 (54 #13347)`] studied this boundary value problem using L^p norms with $p > n$ on $\{(t,x)\colon t = \text{const},\ x \in \Omega\}$. The author uses L^2 norms for u and ∇u. He also proves local (in time) existence and uniqueness theorems, which also hold in the large for $n = 2$ and moreover for $n = 3$ and small initial data. He applies H. Fujitaand T. Kato'smethod [Arch. Rational Mech. Anal. 16 (1964), 269–315; `MR 0166499 (29 #3774)`].

87b:35107
Beals, Michael
Propagation of smoothness for nonlinear second-order strictly hyperbolic differential equations.
Pseudodifferential operators and applications (Notre Dame, Ind., 1984), 21–44, Proc. Sympos. Pure Math., 43, Amer. Math. Soc., Providence, RI, 1985.
Let $u \in H^s(\mathcal{O})$, $\mathcal{O} \subset \mathbb{R}^n$, $s > n/2$, be the solution of a semilinear or quasilinear strictly hyperbolic equation with given singular support on an initial hypersurface or in the past. The singular support of u in the future can be larger than that of the solutions to the corresponding linear problem. These "anomalous singularities" that develop have been shown to arise in two ways: if singularity-bearing characteristics for the linear problem cross, then nonlinear singularities can propagate along all forward characteristics issuing from the crossing point (Rauch-Reed, Lascar, Beals); and, for $n > 2$, if $\Gamma = \{x(s)\}$ is a characteristic that is the projection of two null bicharacteristics $\Gamma_\pm = \{x(s), \pm\xi(s)\}$, and if the corresponding linear solution has wave front set containing $\Gamma_+ \cup \Gamma_-$, then nonlinear singularities can "self-spread" from Γ along all possible characteristics (Beals). Singularities of strength approximately $2s - n/2$ appear in a solution to a nonlinear equation of order greater than two.

For second-order equations anomalous singularities of order $2s - n/2$ do not appear; they are at worst of order roughly $3s - n$: this paper provides complete proofs for the general semilinear equations $p_2(x,D)u = f(x,u)$ and $p_2(x,D)u = f(x,u,Du)$, with f smooth, and an outline of the proof in the quasilinear case. The idea behind the proof bears certain similarities to the arguments used in the study of conormal (Bony, Melrose-Ritter) or stratified (Rauch-Reed) solutions. An algebraic property of $p_2(x,\xi)$ allows estimates on $e_{2k}(D)(f(u))$ for certain microlocally elliptic operators e_{2k} in terms of estimates on $(p_2(x,D))^k u$ in the case of singularities due to self-spreading. Since $(p_2(x,D))^k u \in H^{s-k}(\mathcal{O})$ if $p_2(x,D)u = f(u)$ and $k < s - n/2$, it follows that an extra $s - n/2$ derivatives are estimated where e_{2k} is elliptic. A geometric property of the characteristic set of p_2 then shows that, for both self-spread singularities and those due to characteristic crossings, extra derivatives in characteristic directions of order $s - n/2$ are controlled, allowing the conclusion.

87c:35110
Capozzi, A.; Salvatore, A.
On the equation $Lu = \nabla V(u)$.
Differential problems and the theory of critical points (Bari, 1984), 41–63, Coll. Atti Congr., Pitagora, Bologna, 1984.
The topic is the following problem. Let E be a real Hilbert space. Let $L\colon E \to E$ be a continuous selfadjoint operator, for which 0 is an isolated eigenvalue of finite multiplicity. Let $A\colon E \to E$ be a compact operator such that $A(0) = 0$, $A = \nabla\psi$, where $\psi \in C^1(E,R)$ and $\psi(0) = 0$. Find $u \in E$ such that $Lu = A(u)$ and $u \neq 0$. In other words: Find the nontrivial critical points of the functional $f \in C^1(E,R)$ defined by $f(u) = \frac{1}{2}(Lu,u)_E - \psi(u)$.

A useful review of printed or preprinted papers is made. Special attention is given to two cases satisfying neither the Palais-Smale nor the Cherami condition: the case of "strong resonance"; the case of "bounded potential", for which an unpublished approach is described. It is detailed when the problem is the following nonlinear wave problem: $u_{tt} - u_{xx} = f(x,t,u)$ and $u(x,t) = u(x,t+T)$ for all $t \in \mathbb{R}$ and $x \in [0,\pi]$, $u(0,t) = u(\pi,t) = 0$ for all $t \in \mathbb{R}$, T/π is rational, f is T-periodic in t, $f \in C^1([0,\pi] \times \mathbb{R} \times \mathbb{R}, R)$. Under some other very specific assumptions the existence of solutions is established.

87f:35033
DiPerna, Ronald J.
Weak limits of solutions to nonlinear differential equations.
Physical mathematics and nonlinear partial differential equations (Morgantown, W. Va., 1983), 3–12, Lecture Notes in Pure and Appl. Math., 102, Dekker, New York, 1985.

The author sums up and comments on several papers by Murat, Tartar and himself on compensated compactness. The bottom of p. 7 is disturbing: The author forgets his definition of n by (4) and writes $n\theta < (y,\xi) < (n+1)\theta$ while he means: there exists an integer q such that $q < (y,\xi) < q+\theta$.

87j:35003
Duff, G. F. D.
Singularities, supports and lacunas.
Advances in microlocal analysis (Lucca, 1985), 73–133, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., 168, Reidel, Dordrecht, 1986.
This valuable work is a rich and precise survey of one hundred and four papers about three modern topics. First the author surveys the study of the propagation of singularities for partial and pseudodifferential equations, by the use of Fourier integral operators, wave front sets and microlocal analysis, including the involutive and noninvolutive cases of multiple characteristics. Then the work of C. Feffermanand others on the approximate simultaneous diagonalization of differential operators with variable coefficients is discussed. Finally, for wave equations with variable coefficients, the promising present state of Hadamard's old problem, concerning Huygens' principle of clean cut wave propagation with the support equal to the singular support, is covered.

88b:35156
Hoff, David
Construction of solutions for compressible, isentropic Navier-Stokes equations in one space dimension with nonsmooth initial data.
Proc. Roy. Soc. Edinburgh Sect. A 103 (1986), no. 3-4, 301–315.
Summary: "We prove the global existence of weak solutions for the Cauchy problem for the Navier-Stokes equations for a one-dimensional, isentropic flow when the initial velocity is in L^2 and the initial density is in $L^2 \cap \mathrm{BV}$. Solutions are obtained as limits of approximations obtained by building heuristic jump conditions into a semidiscrete difference scheme. This allows for a rather simple analysis in which pointwise control is achieved through piecewise H^1 and total variation estimates."

88e:35048
Yajima, Kenji
Existence of solutions for Schrödinger evolution equations.
Comm. Math. Phys. 110 (1987), no. 3, 415–426.
The Schrödinger initial value problem is: $i\partial u/\partial t = -\frac{1}{2}\Delta u + V(t,x)u$, $u(0)=u_0$, $t\in[-T,T]$, $x\in\mathbb{R}^n$. Define the Banach space $L^{q,\gamma}(T)=\{W\colon \int_{-T}^{T}[\int_{\mathbb{R}^n}|W(t,x)|^q\,dx]^{\gamma/q}\,dt<\infty\}$; and assume $V\in L^{p,\alpha}(T)+L^{\infty,\beta}(T)$, $p\geq 1$, $\alpha\geq 1$, $\beta>1$, $0\leq 1/\alpha<1-n/2p$, which allows moving singularities of type $|x|^{-2+\varepsilon}$ for $n\geq 4$ and $|x|^{-n/2+\varepsilon}$ for $n\leq 3$, $\varepsilon>0$; then the problem has a unique regular solution for $u_0\in L^2(\mathbb{R}^n)$; $\|u(t)\|=\|u_0\|$ for $|t|\leq T$; the problem generates a strongly continuous unitary propagator. More assumptions give more regularity.

Direct applications of the existing abstract theories lead to rather strong smoothness conditions. Here the characteristic features of Schrödinger equations are taken into account; suitable Banach spaces and estimates are introduced. The results follow from a contraction and from semigroup theory.

88g:35161
Tanaka, Akio
Régularité par rapport au temps des solutions faibles de l'équation de Navier-Stokes.
J. Math. Kyoto Univ. 27 (1987), no. 2, 217–241.
Let Ω be a domain of $\mathbb{R}^3$ whose boundary Γ is compact and of class C^3. Define $V=\{v\in H_0^1(\Omega)\colon \operatorname{div} v=0\}$. Assume that: $u\in L^2(0,T;V)\cap L^\infty(0,T;H)$ and $u\in L^s(0,T;L^r(\Omega))$ for some (r,s) such that $3<r\leq\infty$, $2<s\leq\infty$, $3/r+2/s<1$; either $f\in L^\alpha(0,T;H^{-\beta}(\Omega))$ for some (α,β) such that $0\leq\beta<1$, $\alpha>2/(1-\beta)$, or $f\in C^\gamma([0,T];H^{-1}(\Omega))$ for some γ such that $0<\gamma\leq 1$. Then $u\ (0,T]\to V$ is strongly continuous: $u\in C^0((0,T];V)$.

The increase in the regularity of f increases the regularity of u. In particular: Assume also $\mathcal{O}$ to be an open neighborhood of $(0,T)$ in $\mathbb{C}$ and f to have an analytic continuation $\mathcal{O}\to H^{-1}(\Omega)$; then u has an analytic continuation $\mathcal{O}'\to V$ to some neighborhood $\mathcal{O}'$ of $(0,T)$ in $\mathcal{O}$.

Finally the following assumption is added: $f\ \Omega_0\times\mathcal{O}\to\mathbb{C}^3$ is holomorphic where Ω_0 is some open subset of Ω.

The proof uses a semigroup generated by a selfadjoint, strictly positive definite operator.

88j:35054
Cheng, Kuo-Shung; Lin, Jenn-Tsann

On the elliptic equations $\Delta u = K(x)u^\sigma$ and $\Delta u = K(x)e^{2u}$.
Trans. Amer. Math. Soc. 304 (1987), no. 2, 639–668.
The two equations in the title arise in differential geometry; here they are studied in $\mathbb{R}^n$. The unknown function $u(\cdot)$ is locally bounded; the first equation assumes $u(\cdot) \geq 0$. The given function $K(\cdot) \geq 0$ is bounded HÃ¶lder continuous; $\sigma > 1$ is a given constant. The cases $n \geq 3$, $n = 2$ and $n = 1$ are handled separately. Earlier existence theorems are quoted and five new ones proved. A number of nonexistence theorems constitute the main results. Three of them almost completely answer the following conjecture: Assume $n \geq 3$, $K(x) \geq k(|x|)$ for $x \in \mathbb{R}^n$ and $\int_0^\infty sk(s)\,ds = \infty$; then the first equation would not possess any positive solution in $\mathbb{R}^n$.

88j:35126
Borchers, Wolfgang; Sohr, Hermann
On the semigroup of the Stokes operator for exterior domains in L^q-spaces.
Math. Z. 196 (1987), no. 3, 415–425.
Let Ω be an exterior domain in $\mathbb{R}^n (n \geq 3)$, with compact boundary of class $C^{2+\mu}$ $(0 < \mu <; 1)$. On Ω, the Stokes boundary value problem $-\Delta u + \nabla p = f$, $\operatorname{div} u = 0$, $u\big|_{\partial\Omega} = 0$, is studied for $u \in L^q(\Omega)^n$, $1 < q < \infty$, $p \in L^q_{\rm loc}(\Omega)$. Let P_q be the projection operator of $L^q(\Omega)^n$ onto the subspace $H_q(\Omega)$ of divergence-free fields with zero normal component on $\partial\Omega$; the Stokes operator is $A_q = -P_q\Delta$. It is known that $-A_q$ generates an analytic semigroup $\exp(-tA_q)$ $(t \geq 0)$ in $H_q(\Omega)$. The following new properties are proved: Let $0 < \omega < \pi/2$; for all $\lambda \in \mathbb{C}$ with $|\lambda| > 0$ and $|\arg\lambda| < \omega + \pi/2$, the resolvent $(\lambda I + A_q)^{-1}$ exists and satisfies $|\lambda| \cdot \|(\lambda I + A_q)^{-1}\|_{H_q(\Omega)} \leq c(q,\omega,\Omega)$; $t \mapsto \exp(-tA_q)$ is well defined and analytic for $t \in \mathbb{C}$, $t \neq 0$, and $|\arg t| < \omega$. If $0 < \epsilon < \omega$, $t \in \mathbb{C}$, $t \neq 0$, $|\arg t| < \omega - \epsilon$ and $k = 0, 1, 2, \cdots$, then $|t|^k \cdot \|A_q^k \exp(-tA_q)\|_{H_q(\Omega)} < C(q,\omega,\epsilon,\Omega)$. Moreover $\lim_{t\to\infty} |\exp(-tA_q) \cdot v\|_{H_q(\Omega)} = 0$ for any $v \in H_q(\Omega)$.

88k:35164
Secchi, Paolo
L^2 stability for weak solutions of the Navier-Stokes equations in $\mathbb{R}^3$.
Indiana Univ. Math. J. 36 (1987), no. 3, 685–691.
The following theorem is proved. Let $L^2(\mathbb{R}^3)$ be the space of the square integrable functions $\mathbb{R}^3 \to \mathbb{R}^3$ and $\|\cdot\|$ be its norm. Let $H = \{\varphi \in L^2(\mathbb{R}^3) \operatorname{div}\varphi = 0\}$. Let $H^p = H^p(\mathbb{R}^3)$ be the Sobolev space of order p. Consider the initial value problem for the nonstationary Navier-Stokes equations in the whole space $\mathbb{R}^3$ with a given external force $\in L^1(0,\infty; L^2) \cap L^2(0,\infty; L^2)$. Assume that for some initial velocity $v_0 \in H$ there exists a strong solution $v \in H^1(0,\infty; L^2) \cap L^2(0,\infty; H^2)$. Then for any initial velocity $u_0 \in H$ there exists a suitable weak solution $u \in L^\infty(0,\infty; H) \cap L^2(0,\infty; H^1)$ such that $\|u(t) - v(t)\| \to 0$ as $t \to \infty$. For the construction of suitable weak solutions see a paper by H. Beir ao da Veiga [J. Math. Pures Appl. (9) 64 (1985), no. 3, 321–334;MR 0823407 (87h:35268)].

88m:35037
Hsu, Pei
On the Poisson kernel for the Neumann problem of Schrödinger operators.
J. London Math. Soc. (2) 36 (1987), no. 2, 370–384.
Summary: "Let D be a bounded domain in $\mathbb{R}^d$ $(d \geq 3)$ and let $b(t,x,y)$ be the kernel of the Feynman-Kac semigroup associated with the reflecting Brownian motion $\{X_t\colon\ t \geq 0\}$ and potential V, namely

$$E^x\Big[\exp\Big(\int_0^t V(X_s)\,ds\Big) f(X_t)\Big] = \int_D b(t,x,y)f(y)m(dy).$$

We assume that V is in the Kato class K_d [see M. Aizenman and B. Simon, Comm. Pure Appl. Math. 35 (1982), no. 2, 209–273; MR 0644024 (84a:35062)]. The Poisson kernel studied in this paper is $N_V(x,y) = \int_0^\infty b(t,x,y)\,dt$. In general N_V may be infinite. We show that if $N_V(x,y)$ is finite for one pair of points then it is finite for all $x \neq y$ and there exist two constants c_1, c_2 (depending on D and V) such that $c_1 \leq \|x-y\|^{d-2}N_V(x,y) \leq c_2$. This happens precisely when the spectrum of $H_V = \Delta/2 + V$ under the Neumann boundary condition lies in the negative half-axis. This result is used to discuss the Neumann boundary value problem of H_V. We prove that for any boundary function $f \in L^\alpha(\partial D)$, $\alpha \geq 1$, the problem has a unique weak solution $u_f(x) = \frac{1}{2}\int_{\partial D} N_V(x,y)f(y)\sigma(dy) \in C(D)$ and its growth rate near the boundary can be estimated by $\|f\|_{\alpha,\partial D}$."

89c:35033
Sjöstrand, J.
Tunnel effects for semiclassical Schrödinger operators.
Hyperbolic equations and related topics (Katata/Kyoto, 1984), 347–362, Academic Press, Boston, MA, 1986.

This is a survey of recent results, obtained jointly with B. Helffer, about the semiclassical Schrödinger operator $P = -h^2\Delta + V(x)$ on a compact Riemannian manifold M or on $M = \mathbb{R}^n$. The first ones were already published in the Parts I and II of a joint paper [Comm. Partial Differential Equations 9 (1984), no. 4, 337–408; MR 0740094 (86c:35113); Ann. Inst. H. Poincaré Phys. Théor. 42 (1985), no. 2, 127–212; MR 0798695 (87a:35142)]. The sections are: General abstract results; The case of nondegenerate point wells; Wells formed by submanifolds; Resonances. That last section gives new and very recent results. There $M = \mathbb{R}^n$; it is not certain that P has a selfadjoint realization and even if so that the spectrum of P is discrete near 0. Instead other function spaces will be constructed where again P will have discrete spectrum near 0. The machinery is heavy to build up and is not completely described here.

89d:35141
Greengard, Claude; Thomann, Enrique
On DiPerna-Majda concentration sets for two-dimensional incompressible flow.
Comm. Pure Appl. Math. 41 (1988), no. 3, 295–303.
R. J. DiPerna and A. Majda [J. Amer. Math. Soc. 1 (1988), no. 1, 59–95] studied the nature of the limiting behavior of sequences of solutions of Euler's equations, or appropriate sequences of approximate solutions; in that paper they defined the notion of concentration set. The authors prove that there are no concentration sets of space-time dimension strictly less than one. Therefore approximate solution sequences of the Euler equations either converge strongly or have a concentration set of space-time dimension exactly equal to one, with the set of time coordinates being of positive Lebesgue measure. Finally, an example is given of a bounded sequence of steady solutions of Euler's equations whose weak limit is zero and whose concentration set is uncountable, everywhere dense on the unit square, and of Hausdorff dimension zero.

89g:58203
Ichinose, Wataru
On L^2 well posedness of the Cauchy problem for Schrödinger type equations on the Riemannian manifold and the Maslov theory.
Duke Math. J. 56 (1988), no. 3, 549–588.
Let M be a C^∞ complete Riemann manifold with a countable basis and without boundary. Denote by L^2 the Hilbert space of square-integrable functions on M. In the distribution sense consider the Cauchy problem for the "nonlinear Schrödinger equation" $[-i\partial_t - \frac{1}{2}\Delta + \mathbb{B} + C]u = f$, where Δ denotes the Laplace-Beltrami operator on M, C a given C^∞ function on M, $\mathbb{B}$ a given complexified C^∞ vector field on M and $\mathbb{B}u$ the Lie derivative; the unknown function u and the given one f are continuous functions of $t \in [0,T]$ or $t \in [-T,0]$ $(T>0)$ with values in L^2; the value of u at $t=0$ is given. Denote by $\omega_{\mathbb{B}}$ the 1-form on M defined by $\mathbb{B}$. The following theorem is proved: If the Cauchy problem is well posed, then the following condition holds: $\sup_{\gamma\in\Gamma} |\int_\gamma \mathrm{Re}\,\omega_{\mathbb{B}}| < \infty$, where Γ is the family of all geodesics on M.

The proof deduces from the well-posedness of the problem an estimate of its solutions. Then, assuming the preceding condition not satisfied and using Maslov's construction of asymptotic solutions of the homogeneous problem (i.e. $f=0$), the proof constructs nonhomogeneous problems with solutions contradicting that estimate.

89h:35013
Esser, P.
Sur la réduction de fonctions analytiques qui ne sont pas en involution.
Bull. Soc. Math. Belg. Sér. B 40 (1988), no. 1, 73–79.
The following theorem is proved: Near $\rho \in T^*\mathbb{R}^n$ let f and g be real analytic functions such that $f(\rho) = g(\rho) = 0$, $\{f,g\}(\rho) > 0$, where $\{\cdots\}$ denotes the Poisson bracket; for all $a,b>0$ such that $a+b=1$, there is a unique function $\varphi > 0$, analytic at ρ, such that $\{f\varphi^a, g\varphi^b\} = 1$ near ρ. That theorem extends a result of Kawai and Kashiwara and is similar to a theorem of Hörmander applied to C^∞ functions. It makes possible the microlocal study of the singularities of the solutions of analytic differential operators whose principal symbol is $ep_1^k p_2^l$, where $e(\rho) \neq 0$, $p_1(\rho) = p_2(\rho) = 0$, $\{p_1,p_2\}(\rho) \neq 0$.

89i:35043
Kenig, Carlos E.; Sogge, Christopher D.
A note on unique continuation for Schrödinger's operator.
Proc. Amer. Math. Soc. 103 (1988), no. 2, 543–546.
The following unique continuation theorem is proved for the Schrödinger operator. Let Δ be the Laplace operator; assume $u \in W^p(\mathbb{R}^{n+1})$, $|(i\partial/\partial t + \Delta)u(x,t)| \leq |V(x,t)u(x,t)|$ for some $V \in L^{(n+2)/2}(\mathbb{R}^{n+1})$ and $u=0$ in some half-space of $\mathbb{R}^(n+1)$; then $u=0$ on $\mathbb{R}^{n+1}$. This theorem is a corollary of "uniform Sobolev inequalities" for operators which are the Schrödinger operator plus lower-order terms in x.

89k:35063

Łaba, Izabella
Unique continuation for Schrödinger operators and for higher powers of the Laplacian.
Math. Methods Appl. Sci. 10 (1988), no. 5, 531–542.
The author reviews known unique continuation properties for Schrödinger's operator $-\Delta + V$ and the motivation for them. Using a Carleman-type inequality proved by D. Jerison and C. E. Kenig [Ann. of Math. (2) 121 (1985), no. 3, 463–494; MR 0794370 (87a:35058)] and following a suggestion they made, she proves a unique continuation property for $-\Delta^\mu + V$.

89m:35182
Miyakawa, Tetsuro; Sohr, Hermann
On energy inequality, smoothness and large time behavior in L^2 for weak solutions of the Navier-Stokes equations in exterior domains.
Math. Z. 199 (1988), no. 4, 455–478.
Let Ω be an exterior domain in $\mathbb{R}^n$ ($n = 3, 4$) with smooth $\partial\Omega$. On $\Omega \times [0, \infty)$ consider the Navier-Stokes problem: $u_t - \Delta u + u \cdot \nabla u + \nabla p = f$ and $\nabla u = 0$ in $\Omega \times (0, \infty)$, $u = 0$ on $\partial\Omega \times (0, \infty)$, $u(x, 0) = a(x)$, with u and p unknown, a and f given. Introduce the energy inequality

$$\|u(t)\|_2^2 + 2\int_{(s,t)} \|\nabla u\|_2^2 d\tau \leq \|u(s)\|_2^2 + 2\int_{(s,t)} (f, u) d\tau$$

for $s = 0$, a.e. $s > 0$, and all $t \geq s$. Denote by X_2 the L^2-closure of the set of the smooth solenoidals vector fields with compact support in Ω. Theorem 1: Let $n = 3$ or 4 and suppose that $f \in L^2(0, T; X_2)$ for all $T > 0$; then there is a weak solution of the Navier-Stokes problem satisfying the energy inequality. If in addition $f \in L^1(0, \infty; X_2)$, then any weak solution satisfying that inequality is such that $\lim_{t\to\infty} \|u(t)\|_2 = 0$; if moreover f is smooth enough and $f \in L^2(0, \infty; X_2)$, then there is a $T_0 > 0$ depending only on a, f, Ω such that u is a classical solution in the interval (T_0, ∞).

Assume now $n = 3$ and w to be a stationary solution of Finn's type; Theorem 2 similarly gives solutions u such that $\lim_{t\to\infty} \|u(t) - w(t)\|_2 = 0$.

90a:35079
Maz'ja, Vladimir Gilelevič; Rossmann, Jürgen
Über die Asymptotik der Lösungen elliptischer Randwertaufgaben in der Umgebung von Kanten.
Math. Nachr. 138 (1988), 27–53.
Summary: "The topic is the behaviour of the solutions of elliptic boundary value problems in the neighborhood of edges. Here, the solution and the right-hand sides are assumed to be functions from suitable weighted Sobolev spaces. Under certain assumptions on the boundary values and on the right-hand sides we show that the solution u differs by some 'singular terms' from a 'regular' one. Moreover an exact characterization of the coefficients of the asymptotics is given. In contrast to previous papers there is no additional assumption about the smoothness of the right-hand sides in the direction of the edge."

90a:41027
Dostál, Miloš; Gaveau, Bernard
Comportement asymptotique d'intégrales de Fourier-Laplace avec points critiques dégénérés de la phase de type 1 ou 2.
C. R. Acad. Sci. Paris Sér. I Math. 307 (1988), no. 20, 981–984.
Let $I(R) = \int_{[0,R]^n} \exp[i\lambda\varphi(x)]\, dx$, where φ is a polynomial of degree d. Assume $d > n$, the coefficients of the principal part of φ to be ≥ 0, the coefficients of $x_1^d, \cdots, x_n^d$ to be > 0. Then $\lim_{R\to\infty} I(R) = -q^n \int_{[0,\infty)^n} \exp[\lambda\Phi(x)]\, dx$, where $q = \exp[i\pi/2d]$.

Now let $J(\lambda) = \int_{[0,\infty]^n} \exp[\lambda\Phi(x)]\, dx$, where λ is large, Φ is a polynomial of degree d, $\Phi(x) = \sum_\mu a_\mu x^\mu$, $\operatorname{Re} a_\mu \leq 0$, $\Phi(0) = 0$ and 0 is a critical point of Φ. Then the asymptotic behaviour of $J(\lambda)$ is explicitly given in the two simplest cases. Other cases will be given in another paper.

90c:35162
Deuring, P.; von Wahl, W.; Weidemaier, P.
Das lineare Stokes-System in $\mathbb{R}^3$. I. Vorlesungen über das Innenraumproblem.
Bayreuth. Math. Schr. No. 27 (1988), vi+252 pp.
This is the first of a series of monographs publishing von Wahl's lectures, whose motivation is his opinion that O. A. Ladyzhenskayahas only suggested the proofs of her assertions in her well-known work The mathematical theory of viscous incompressible flow [Gordon & Breach, New York, 1963; MR 0155093 (27 #5034b); second edition, 1969; MR 0254401 (40 #7610)]. In his excellent review of that second edition, R. Finnwrites: "This

volume exhibits the spirit of modern applied mathematics in one of its best and most fruitful forms"; he also makes many accurate critical remarks.

The general topic of these monographs is Navier-Stokes equations. The first monograph under review here deals with only the time independent linear Stokes equations: $-\nu\Delta u_i + \partial\pi/\partial x_i = f_i$, $\sum_i \partial u_i/\partial x_i = 0$ in A, where A is a domain of $\mathbb{R}^3$ such that ∂A is smooth and either A or $\mathbb{R}^3 \setminus A$ is bounded; $\nu = \text{const}$; $i = 1, 2, 3$; the functions f_i are given on A. The problem to be solved is the following: Determine u_i and π on $\overline{A}$ when, on ∂A, the functions u_i are given, such that $\iint_{\partial A} u_1 dx_2 \wedge dx_3 + u_2 dx_3 \wedge dx_1 + u_3 dx_1 \wedge dx_2 = 0$. The following is carefully proved: under suitable assumptions this problem has a unique solution, differentiable in A, such that the u_i are continuous on $\overline{A}$; if A is bounded, then, under other assumptions, the second derivatives of u_i and the first derivatives of π belong to $L_p(A)$. Ladyzhenskaya's integral equation method is applied; it makes use of Oseen and Odqvist potentials, but those two names are not quoted. Many other mathematical techniques are used and clearly detailed or quoted. This monograph is said to be written for students—I would say for very studious students, able to master some 58 lemmata and 18 theorems.

`90c:35190`
Mu, Mu
Necessary and sufficient conditions for existence of global classical solutions of two-dimensional Euler equations in time-dependent domain.
Kexue Tongbao (English Ed.) 33 (1988), no. 15, 1295–1299.
Euler equations contain Coriolis' force. The time-dependent domain $\Omega(t)$ is bounded simply connected; $\Omega(t) = \{(x_1, x_2, t)\colon \gamma(x_1, x_2, t) < 0\}$, where $t \geq 0$, $\gamma \in C^\infty(\mathbb{R}^2 \times \overline{R}_+)$; the line element of $\partial\Omega(t)$ is dl_t. Using a paper he published in Chinese [Acta Math. Sci. (Chinese) 6 (1986), 201–218; per bibl.], the author proves the equivalence of the following three assertions: existence of a global classical solution; $\int_{\partial\Omega(t)} [(\partial\gamma/\partial t)/|\nabla_x \gamma|] dl_t = 0$ for all $t > 0$; existence of a measure-preserving mapping $\overline{\Omega}(0) \mapsto \overline{\Omega}(t)$ varying smoothly with t. A paper by H. Kozono [J. Differential Equations 57 (1985), no. 2, 275–302;`MR 0788281 (86i:35120)`] is discussed.

`90g:35127`
Deuring, Paul; von Wahl, Wolf
Das lineare Stokes-System in $\mathbb{R}^3$. II. Das Außenraumproblem.
Bayreuth. Math. Schr. No. 28 (1989), 1–109.
This work is the continuation of Part I [same journal No. 27 (1988); `MR 0973249 (90c:35162)`]. The following results are carefully established. The problem $-\nu \cdot \Delta \overrightarrow{u} + \nabla\pi = \overrightarrow{f}$, div $\overrightarrow{u} = 0$ in $\mathbb{R}^3 \setminus \overline{\Omega}$, $\overrightarrow{u}|_{\partial\Omega} = \overrightarrow{a}$ has a unique solution. The data are: the constant ν, the bounded domain $\Omega \subset \mathbb{R}^3$ ($\partial\Omega$ being $C^{2,\gamma}$), $\overrightarrow{f} \in L_1(\mathbb{R}^3 \setminus \overline{\Omega})^3 \cap L_p(\mathbb{R}^3 \setminus \overline{\Omega})$ and $\overrightarrow{a} \in W^{2-1/p,p}(\partial\Omega)^3$, where $\frac{3}{2} < p < \infty$; the unknowns are u and π; $u \in L_s(\mathbb{R}^3 \setminus \overline{\Omega})^3$ if $3 < s < \infty$; π, the first and second derivatives of $\overrightarrow{u}$ belong to L_p. The proof of that theorem uses potentials and an integral equation. Moreover, the Helmholtz decomposition and, having chosen $a = 0$ and $p = 2$, arguments of that proof lead to the definition of a "Stokes operator" and to its properties; that operator is selfadjoint, which will later be applied to the linear Navier-Stokes system.

`90h:35032`
Bloom, C. O.; Kazarinoff, N. D.
Energy decay for hyperbolic systems of second-order equations.
J. Math. Anal. Appl. 132 (1988), no. 1, 13–38.
Let V be a simply connected, unbounded domain of $\mathbb{R}^3$, with piecewise smooth, star-shaped boundary ∂V. In $(V \cup \partial V) \times [0, \infty)$ an initial-boundary value problem is defined by a system of m second-order differential equations in m unknowns; this system is hyperbolic; the boundary data are Cauchy initial data and Dirichlet data on ∂V.

That problem is studied in three cases. A divergence identity gives rise to two basic "energy identities". The total energy is bounded from above by a multiple of its initial value. A "domain of dependence theorem" is asserted: the solution has compact support in $V \cup \partial V$ for all $t \in (0, \infty)$. Existence and uniqueness theorems can be obtained. An appropriate definition of "the local energy $\mathcal{E}(\cdot)$ of a solution" is given; it depends on $t \in [0, \infty)$. The main theorem is the following assertion: In the three cases under consideration $\mathcal{E}(t) \leq \text{const} \cdot t^{-c} \xi(0)$, where C is a strictly positive constant.

`90k:35199`
Calderón, Calixto P.
Existence of weak solutions for the Navier-Stokes equations with initial data in L^p.
Trans. Amer. Math. Soc. 318 (1990), no. 1, 179–200.
Summary: "The existence of weak solutions to the Navier-Stokes equations for an infinite cylinder with initial data in L^p is considered in this paper. We study the case of initial data in $L^p(\mathbb{R}^n)$, $2 < p < n$, and

$n = 3, 4$. An existence theorem is proved covering these important cases and, therefore, the 'gap' between the Hopf-Leray theory ($p = 2$) and that of Fabes-Jones-RiviÃ¨re ($p > n$) is bridged. The existence theorem gives a new method of constructing global solutions. The cases $p = n$ are treated at the end of the paper."

For an addendum see the following review.

90k:35200
Calderón, Calixto P.
Addendum to the paper: "Existence of weak solutions for the Navier-Stokes equations with initial data in L^p" [Trans. Amer. Math. Soc. 318 (1990), no. 1, 179–200; MR 0968416 ((90k:35199)].
Trans. Amer. Math. Soc. 318 (1990), no. 1, 201–207.
Summary: "We consider the existence of global weak solutions for the Navier-Stokes equations in the infinite cylinder $\mathbb{R}^n \times \mathbf{R}_+$ with initial data in L^r, $n \geq 3$, $1 < r < \infty$. An embedding theorem as well as related initial value problems are also studied, thus completing results in the paper cited in the heading [see the preceding review]."

91e:58177
Berhanu, S.
Microlocal Holmgren's theorem for a class of hypo-analytic structures.
Trans. Amer. Math. Soc. 323 (1991), no. 1, 51–64.
Summary: "In 1981 J. Sjöstrand [Comm. Partial Differential Equations 6 (1981), no. 5, 499–567; MR 0613851 (82k:35011)] gave a simpler proof of a result of Schapira concerning a microlocal version of Holmgren's theorem for real analytic data. Inspired by Sjöstrand's proof, in this paper we extend Schapira's result to a certain class of hypoanalytic structures. The paper is organized as follows: In Section 2 we discuss the Cauchy-Kovalevskaya theorem for maximal hypoanalytic structures. In Section 3 we introduce a class of hypoanalytic structures which we call real hypoanalytic, give a statement of the main theorem of this article and derive two corollaries. A lemma is included in the same section and is used in the proof of the main theorem, which appears in Section 4."

91j:35226
Titi, Edriss S.
On approximate inertial manifolds to the Navier-Stokes equations.
J. Math. Anal. Appl. 149 (1990), no. 2, 540–557.
Section 1 of this paper reviews the theory of the 2-dimensional Navier-Stokes equations with initial data and either the homogeneous Dirichlet boundary condition or the double periodicity condition. Paragraph 2.1 recalls the concepts of inertial manifold (IM) and approximate IM. Theorem 2.1 states results of C. Foias, O. Manley and R. Temam [C. R. Acad. Sci. Paris Sér. I Math. 305 (1987), no. 11, 497–500; MR 0916319 (88j:76026); RAIRO Modél. Math. Anal. Numér. 22 (1988), no. 1, 93–118; MR 0934703 (89h:76022)]; to indicate their importance an illustrative example is presented. Paragraph 2.2 recalls the analytic manifold M^s due to Foias, J.-C. Saut and Temam [Foias and Temam, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) 5 (1978), no. 1, 28–63; MR 0481645 (58 #1749); Foias and Saut, ibid. Cl. Sci. (4) 10 (1983), no. 1, 169–177; MR 0713114 (85h:35171)]. It is an approximate IM, which is a C-analytic manifold; the orbits in the universal (global) attractor are closer to M^s than to the finite-dimensional manifold M_0 introduced by Foias, Manley and Temam; M_s leads to sharp inequalities. But M_0 is explicitly known, whereas M_s is constructed by the contraction principle. For that reason paragraph 2.3 gives explicit approximations of M^s that approximate the universal attractor as well as M^s does. Those results can be used for real computations. Several of them can be extended to the 3-dimensional Navier-Stokes equations.

The error estimates are similar to those obtained in Temam's recent asymptotic approximations [C. R. Acad. Sci. Paris Sér. II Méc. Phys. Chim. Sci. Univers Sci. Terre 306 (1988), no. 6, 399–402; MR 0979153 (90a:76067)].

91m:65059
Connor, J. N. L.
Practical methods for the uniform asymptotic evaluation of oscillating integrals with several coalescing saddle points. Asymptotic and computational analysis (Winnipeg, MB, 1989), 137–173, Lecture Notes in Pure and Appl. Math., 124, Dekker, New York, 1990.
The integrals considered are $I(\alpha) = \int_{-\infty}^{\infty} g(t) \exp[i f(\alpha; t)/\hbar] dt$, $\hbar \to 0$, where f and g are analytic functions; $f(\alpha; t)$ is real and $\alpha \in \mathbb{R}^p$. The behaviour of $I(\alpha)$ does not change if $I(\alpha)$ is computed on a neighbourhood of the saddle points of the function $t \mapsto f(\alpha; t)$. Those saddle points depend on α and can coalesce as α varies. The behaviour of $I(\alpha)$ can be expressed in terms of "cuspoid" canonical integrals and of their derivatives.

For n coalescing saddle points the "cuspoid" canonical integral is

$$C_{n+1}(a)=\int_{-\infty}^{\infty}\exp[i(u^{n+1}+a_1u+\cdots+a_{n-1}u^{n-1})]du.$$

For $n=2$ the result is easy; $C_3(a_1)$ is Airy's function. The purpose of the paper is to discuss the two practical techniques that have been applied and then to outline their results in the cases $n=3$ and $n=4$. The first technique performs the numerical quadrature along a convenient path in the complex u plane. The second technique computes $C_{n+1}(a)$ by numerically integrating a set of differential equations satisfied by $C_{n+1}(a)$. Some of the results are shown by a contour plot and thirteen perspective plots. There are 70 references.

92b:35121
Amick, Charles J.
On the asymptotic form of Navier-Stokes flow past a body in the plane.
J. Differential Equations 91 (1991), no. 1, 149–167.
Let $\Omega=\mathbb{C}^2\setminus K$, where K is a compact set with smooth boundary; let w_∞ be a nonzero constant vector. Consider on Ω the solutions w of the stationary Navier-Stokes equations such that $w=0$ on $\partial\Omega$ and $w\to w_\infty$ at infinity, whose Dirichlet norms are finite. The asymptotic behavior of those solutions is known if their decay is $|w(x,y)-w_\infty|=O(r^{-1/4-\epsilon})$ for $r^2=x^2+y^2\to\infty$. The main theorem asserts that those solutions have the faster decay $|w(x,y)-w_\infty|=O(r^{-1/2+\epsilon})$; hence by quoted results the faster decay $|w(x,y)-w_\infty|=O(r^{-1/2})$. That main theorem follows from ingenious estimates of the decay of the vorticity ω, of its product $\phi\cdot\omega$ by the Stokes stream function ϕ and from preliminary estimates of the decay of $w(x,y)-w_\infty$. The references are about the Navier-Stokes flows in the plane and also in the space.

92c:35047
Han, Zheng-Chao
Asymptotic approach to singular solutions for nonlinear elliptic equations involving critical Sobolev exponent.
Ann. Inst. H. Poincaré Anal. Non Linéaire 8 (1991), no. 2, 159–174.
The author establishes results related to a Brézis-Peletier conjecture about the blowing up of solutions [H. Brézisand L. A. Peletier, in Partial differential equations and the calculus of variations, Vol. I, 149–192, Birkhäuser Boston, Boston, MA, 1989; MR 1034005 (91a:35030)]. Let Ω be a smooth bounded domain in $\mathbb{R}^N$ with $N\geq 3$. Denote by $p=(N+2)/(N-2)$ the critical exponent; let u_ε be a subcritical solution; i.e. let $\varepsilon>0$, $-\Delta u_\varepsilon=N(N-2)u_\varepsilon^{p-\varepsilon}$ in Ω, $u_\varepsilon>0$ in Ω, $u_\varepsilon=0$ on $\partial\Omega$. The existence of u_ε is well known. Assume that $\lim_{\varepsilon\to 0}\int_\Omega|\nabla u_\varepsilon|^2/\|u_\varepsilon\|^2_{L^{p+1-\varepsilon}(\Omega)}=S_N$, where $S_N=\pi N(N-2)[\Gamma(N/2)/\Gamma(N)]$ is the best Sobolev constant in $\mathbb{R}^N$. After passing to a subsequence, when $\varepsilon\to 0$, there exists $x_0\in\Omega$ such that $\lim u_\varepsilon=0$ in $C^1(\Omega\setminus\{x_0\})$ and

$$\lim|\nabla u_\varepsilon|^2=N(N-2)[S_N/N(N-2)]^{N/2}\delta_{x_0}$$

in the sense of distributions; δ is the Dirac distribution; x_0 is a critical point of an explicitly given function; $\lim\varepsilon\|u_\varepsilon\|^2_{L^\infty(\Omega)}$ and $\lim\varepsilon^{-1/2}u_\varepsilon$ are also explicitly given functions.

A similar result is established when u_ε is a solution of the problem $-\Delta u_\varepsilon=N(N-2)u_\varepsilon^p+\varepsilon u_\varepsilon$ in Ω, $u_\varepsilon>0$ in Ω, $u_\varepsilon=0$ on $\partial\Omega$.

92e:35060
Yajima, Kenji
On smoothing property of Schrödinger propagators.
Functional-analytic methods for partial differential equations (Tokyo, 1989), 20–35, Lecture Notes in Math., 1450, Springer, Berlin, 1990.
The Schrödinger equation under consideration is of the form $i\partial_t u=\frac{1}{2}\sum_j[i\partial_j+A_j(t,x)]^2u+V(t,x)$, $j=1,\cdots,n$, $t\in\mathbb{R}$, $x\in\mathbb{R}^n$, where ∂_j is the component of the gradient ∂_x and both the vector-potential $A=(A_1,\cdots,A_n)$ and the scalar potential V are real-valued. Let $B_{jk}=\partial_jA_k-\partial_kA_j$ and assume that: $|\partial_x^\alpha A(t,x)|+|\partial_x^\alpha\partial_tA(t,x)|\leq C_\alpha$ for any multi-index α; $|\partial_x^\alpha B_{jk}(t,x)|\leq C_\alpha(1+|x|)^{-1-\varepsilon}$ for some $\varepsilon>0$; $\partial_x^\alpha V$ is continuous for any α, and $|\partial_x^\alpha V(t,x)|\leq C_\alpha$ for $|\alpha|\geq 2$. Define

$$\|f\|^2_{\Sigma(2)}=\sum_{|\alpha+\beta|\leq 2}\|x^\alpha\partial_x^\beta f\|^2,$$

where $\|\cdot\|$ is the norm in $L^2(\mathbb{R}^n)$, and $\Sigma(2)=\{f\in L^2(\mathbb{R}^n)\colon \|f\|_{\Sigma(2)}<\infty\}$. A preceding paper proved the existence in $L^2(\mathbb{R}^n)$ of a unitary operator $U(t,s)$, depending on t and $s\in\mathbb{R}$, having the following property: Let $u_0\in C^1(\mathbb{R},L^2(\mathbb{R}^n))\cap C^0(\mathbb{R},\Sigma(2))$; then the Schrödinger equation has a unique solution $u(\cdot)=U(\cdot,s)u_0$ satisfying the initial condition $u(s)=u_0$.

Now let $\mathcal{S}(\mathbb{R}^n)$ be the space of rapidly decreasing functions and $\langle x\rangle = (1+x^2)^{1/2}$, $\langle D\rangle = (1-\Delta)^{1/2}$. The author proves the following theorems. Theorem 1: Let $T>0$ be small, $\mu > 1/2$, $\rho \geq 0$. Then there exists a constant $C_{\rho,\mu}>0$ such that for $s\in\mathbb{R}$, $\int_{s-T}^{s+T} \|\langle x\rangle^{-\mu-\rho}\langle D\rangle^{\rho} U(t,s)f\|^2\,dt \leq C_{\rho\mu}\|\langle D\rangle^{\rho-1/2} f\|^2$, $f\in\mathcal{S}(\mathbb{R}^n)$. Theorem 2: Let $T>0$ be small, $p\geq 2$, $0\leq 2/\theta = 2\sigma + n(\frac{1}{2}-1/p) < 1$ and $\rho\in\mathbb{R}$. Then there exists a constant $C_{p\rho\sigma}>0$ such that

$$\Big[\int_{s-T}^{s+T}\Big\{\int_{\mathbb{R}^n} |\langle x\rangle^{-2\sigma-|\rho|}\langle D\rangle^{p+\sigma}U(t,s)f(x)|^p\,dx\Big\}^{\theta/p}dt\Big]^{1/\theta} \leq C_{p\rho\sigma}\|\langle D\rangle^{p} f\|,$$

for $s\in\mathbb{R}$, $f\in\mathcal{S}(\mathbb{R}^n)$. A maximal inequality and a summability theorem follow from Theorem 1.

92g:35041
Nakai, Mitsuru
Continuity of solutions of Schrödinger equations.
Math. Proc. Cambridge Philos. Soc. 110 (1991), no. 3, 581–597.
Let m be the Lebesgue measure on $\mathbb{R}^d$, where $d\geq 2$. Let μ be a signed Radon measure on an open subset Ω of $\mathbb{R}^d$; let $|\mu|$ be its total variation measure; let $|\mu|_s = |\mu| - (d|\mu|/dm)m$ be its m-singular part. Let $N(x,y)$ be the Newtonian kernel on $\mathbb{R}^d$, that is, $-\log|x-y|$ for $d=2$ and $|x-y|^{2-d}$ for $d\geq 3$. The measure μ is said to be of Kato class if, for every y in Ω: $\lim_{r\downarrow 0}(\sup_{|x-y|<r}\int_{|\xi-y|<r} N(x,\xi)\,d|\mu|(\xi)) = 0$.

The time-independent Schrödinger equation under consideration is $(-\Delta+\mu)u = 0$. If the function u belongs to $L^1_{\mathrm{loc}}(\Omega,|\mu|+m)$ and satisfies $-\int_\Omega u(\xi)\Delta\varphi(\xi)\,dm(\xi) + \int_\Omega u(\xi)\varphi(\xi)\,d\mu(\xi) = 0$ for every test function φ in $C_0^\infty(\Omega)$, then u is said to be a distributional solution on Ω of the above Schrödinger equation. Assume there are a subset X of Ω and a function $\tilde{u}$ in $L^1_{\mathrm{loc}}(\Omega,|\mu|+m)$, continuous at each point of X as a function on Ω, and such that $u=\tilde{u}$, $(|\mu|+m)$-a.e. on Ω; then u is said to be continuous on X. This paper gives an elementary proof to the following theorem: A distributional solution u on Ω is continuous on Ω if and only if u is continuous on Ω except for a subset of Ω of $|\mu|_s$-measure zero. Corollary 1: A distributional solution u on Ω is continuous on Ω if and only if u is continuous on $\operatorname{supp}|\mu|_s$. Corollary 2 (Aizenmann and Simon, 1982): If μ is of Kato class and is m-absolutely continuous, then every distributional solution is continuous. Corollary 3: Any distributional solution on Ω which is continuous on Ω except for a discrete subset of Ω is continuous on the whole of Ω.

92h:35196
Meftah, Mokhtar
Conditions suffisantes pour la compacité de la résolvante d'un opérateur de Schrödinger avec un champ magnétique.
J. Math. Kyoto Univ. 31 (1991), no. 3, 875–880.
The Schrödinger operator on $L^2(\mathbb{R}^n)$ under consideration is $H(\mathfrak{a})+V$; the magnetic potential $\mathfrak{a} = (a_1,\cdots,a_n)$ is real and $a_j\in\mathbb{C}^\infty(\mathbb{R}^n)$; $H(\mathfrak{a}) = \sum_{j=1}^n (D_j - a_j)^2$, where $D_j = -i\partial_{x_j}$; the electric potential V is real, is semibounded from below and can be expressed as $V = \sum_{j=1}^p V_j^2$, where $V_j\in C^\infty(\mathbb{R}^n)$. Sufficient conditions for the compactness of the resolvent of the operator $H(\mathfrak{a})+V$ are given. The arguments and the results extend those of B. Helffer and A. Mohamed [Ann. Inst. Fourier (Grenoble) 38 (1988), no. 2, 95–112; MR 0949012 (90d:35215)].