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Bayesian modelling applied to archaeology for users of Chronomodel and ArcheoPhase

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Plan

Bayesian approach to Interpreting Archaeological Data

The statistical modelling within the Bayesian framework is widely used by archaeologists :

- 1988 Naylor , J . C. and Smith, A. F. M.
- 1990 Buck C.E.
- 1994 Christen, J. A.
- etc

Examples

- Bayesian interpretation of 14C results , calibration of radiocarbon results.
- Constructing a calibration curve.
  to convert a measurement into calendar date
- Bayesian models for relative archaeological chronology building.

Softwares

1. BCal is an on-line Bayesian radiocarbon calibration tool.

Ref : Buck C.E., Christen J.A. and James G.N. (1999). BCal : an online Bayesian radiocarbon calibration tool. Internet Archaeology, 7

2. Oxcal provides radiocarbon calibration and analysis of archaeological and environmental chronological information.

Ref : Bronk Ramsey, C. (1995). Radiocarbon calibration and analysis of stratigraphy : The OxCal program. Radiocarbon, 37(2), 425-430.

3. Chronomodel :

Ref Ph Lanos, A. Philippe (2016) Hierarchical Bayesian modeling for combining Dates in archaeological context. Journal de la SFdS.

Observations

Each dating method provides a measurement M, which may represent :

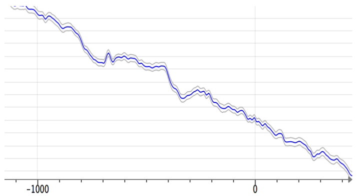
- a 14C age,
- a paleodose measurement in TL/OSL,
- an inclination, a declination or an intensity of the geomagnetic field

Relation with calendar date

$$M = g(\theta) + \epsilon$$

where

- θ is the calendar time
- g is a calibration function which relates the measurement to θ



Radiocarbon *IntCal14*

Archaeological information

After the archaeological excavations, prior information is available on the dates.

Examples :

- Dated archaeological artefacts are contemporary
- Stratigraphic Information which induces an order on the dates.
- the differences between two dates is known (possibly with an uncertainty).
- *Terminus Post Quem/ Terminus Ante Quem*
- etc

Bayesian statistics

- Observations $M_1, M_2, \ldots M_N$ whose the distribution depends on unknown parameter $f(M_1, \ldots M_n|\theta)$
- θ is the unknown parameters. We build a prior distribution on θ : $\pi(\theta)$

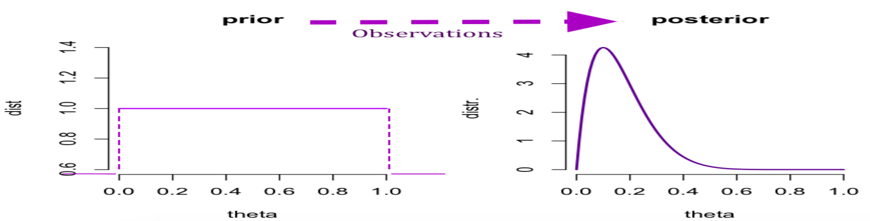
Example

- M_i : 14C ages done on artefact.
- θ : calendar date of artefact

Bayes Formula

The posterior distribution :

$$\pi(\theta|M_1, \ldots M_n) \propto f(M_1, \ldots M_n|\theta) \times \pi(\theta)$$

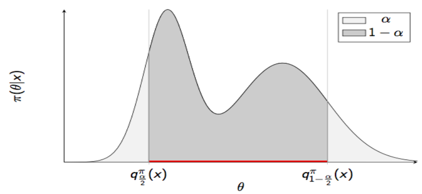


Bayesian inference

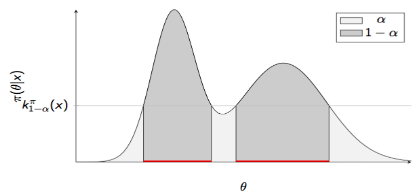
From the posterior distribution, we calculate

- Confidence region :

Credible interval



HPD region



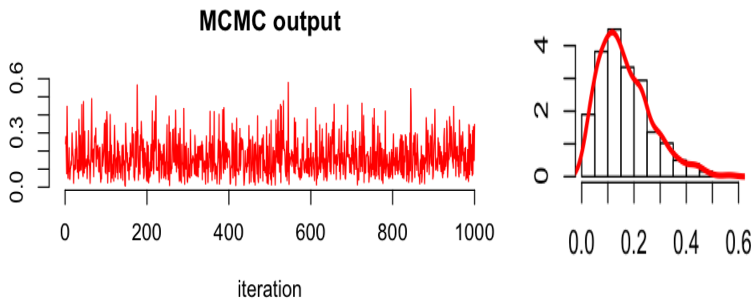
$$P(\theta \in \text{IC or HPD } |M_1, \ldots M_n) = 1 - \alpha$$

- Pointwise Estimates :
  - Mean of the posterior
  - Mode of the posterior

Numerical approximation

Problem
An explicit form of the posterior distribution $\pi(\theta|M_1, ...M_n)$ is not available

Solution
We simulate a sample using MCMC algorithm from the posterior distribution



Plan

Introduction

Chronomodel

RChronoModel - ArcheoPhase

Contribution

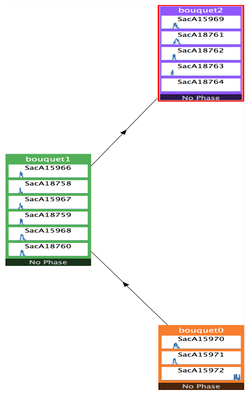
We propose Bayesian tools for constructing chronological scenarios in archaeology.

1. The key point is the EVENT Model : a robust model for combining dates

Definition of the target events :

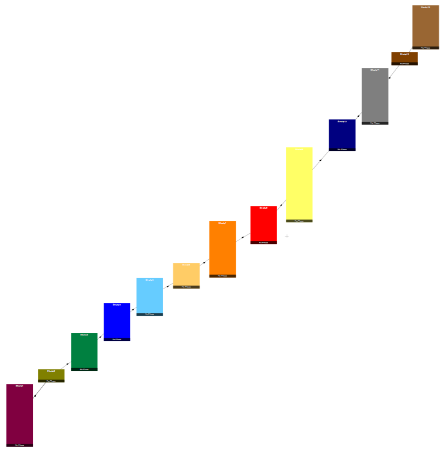
- we choose the group of dated artefacts that are related the target event.
- ⇝ Characterize the date of a target event from the combination of the dates of contemporaneous artefacts.

2. We construct a chronology (= collection of dates) of target events taking into account temporal relationship between the dates of target events



Volcanic eruptions

- **Target Event :** Eruptive period with flow deposits
- **Dated artefacts :** organic samples found in a flow deposit are dated by 14C.
- **Prior information** Stratigraphic constraint on deposits



Chronomodel restrictions

- Each event contains at least one measurement.
- Each measurement is associated to one (and only one) target event.

Definition of the Event Model

Lanos & Philippe (2016)

1. The target event is defined by
   - n measurements.
     For each $i = 1, ..., n$ the measurement M_i is done on archaeological artefact with unknown calendar date t_i :
2. We want to estimate θ. the date of the target event.

The model is

$$M_i = g_i(t_i) + \epsilon_i \qquad \epsilon_i \text{ represents the experimental and calibration error}$$
$$t_i = \theta + \lambda_i$$
$$\theta \sim \text{Uniform } (T) \text{ the study period}$$

Assumptions on λ_i :

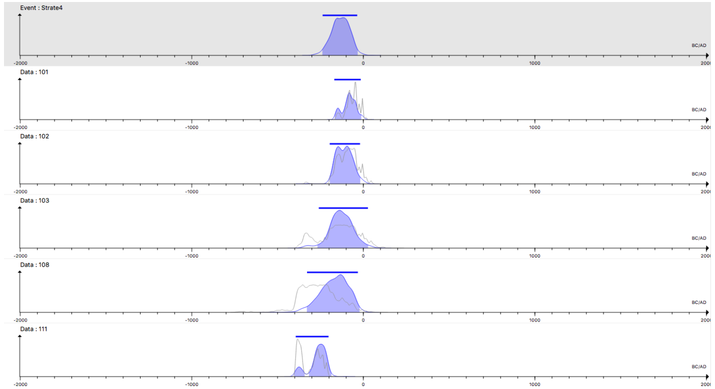
λ_i represents the difference between the date of artifacts t_i and the target event θ This error is external to the laboratory.

$$\lambda_i \sim_{ind} \mathcal{N}(0, \sigma_i^2)$$

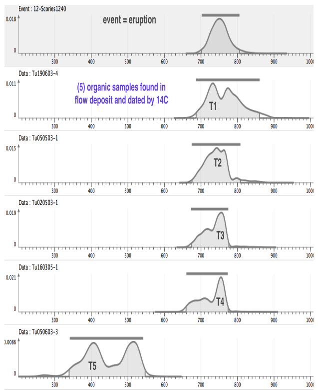
$\rightsquigarrow$ σ_i is the central parameter to ensure the robustness

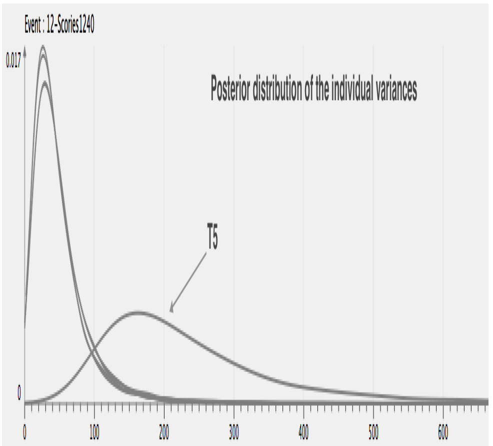
Focus on one pyroclastic flow

- Target event : eruption $[\theta]$
- 5 organic samples found in flow deposit are dated by 14C $[t_1, ..., , t_5]$



Robustness of event model





- the posterior density of date of the target Event remains almost insensitive to the outlier.
- We do not have to choose specific tools for rejecting outlying data.

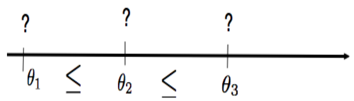
Chronologies of K target events

- We want to estimate $\theta_1, ...\theta_K$ the calendar dates of target events.

Prior information on the dates of the target event

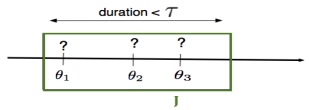
1. The stratigraphic constraints.

$\rightsquigarrow$ a partial order on $(\theta_1, ...\theta_K) := \vartheta \subset T^K$



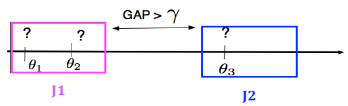
2. Duration information :

$\max_{j \in J} \theta_j - \min_{j \in J} \theta_j \leq \tau$ where τ is known



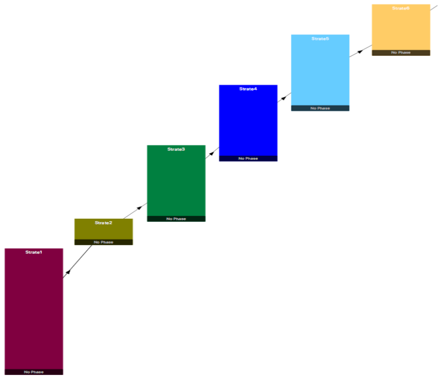
3. Hiatus information :

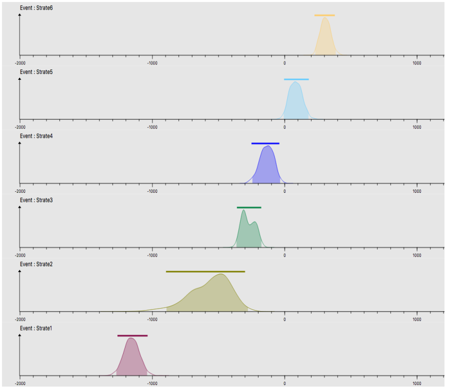
J_1, J_2 two groups, $\min_{j \in J_2} \theta_j - \max_{j \in J_1} \theta_j \geq \gamma$ where γ is known



Chronology of Volcanic eruptions

6 pyroclastic flows from volcano dated by 14C ⇝ 6 ordered target events
$S = \{\vartheta : \theta_1 \leq \ldots \leq \theta_6\}$





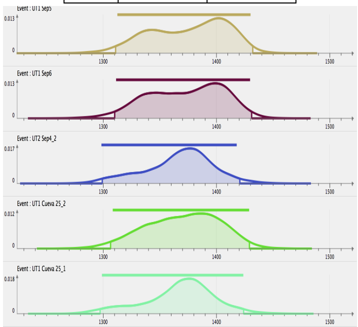
Maya city with information on occupation time

Prior information on the archaeological phase :
The occupation time is smaller than 50 years.

Comparison : HPD regions and posterior densites

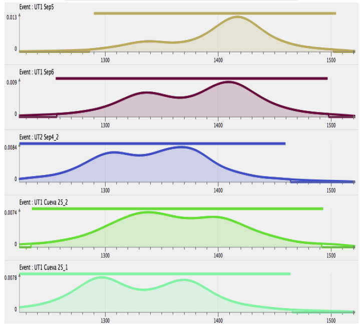
Prior information on the duration

θ_1	1309	1433
θ_2	1308	1430
θ_3	1299	1423
θ_4	1305	1429
θ_5	1297	1425

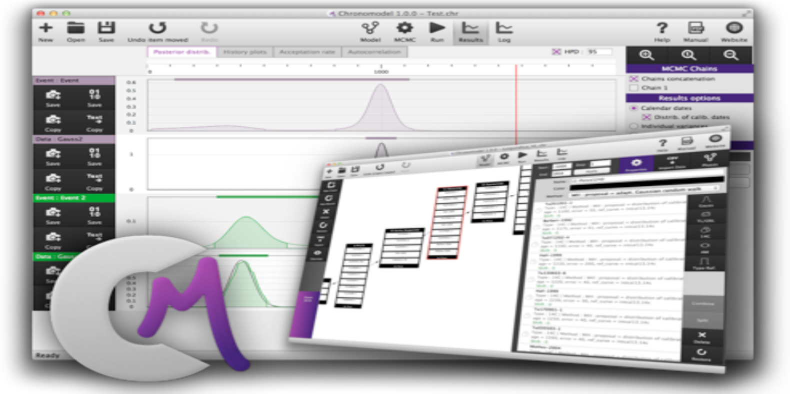


without prior information

θ_1	1284	1506
θ_2	1253	1502
θ_3	1213	1469
θ_4	1230	1497
θ_5	1192	1469



Software : ChronoModel



Plan

Contribution

A R package with its web interface ArcheoPhase :

- Compatible with Oxcal or Chronomodel.
- The inputs are MCMC samples generated by both softwares.

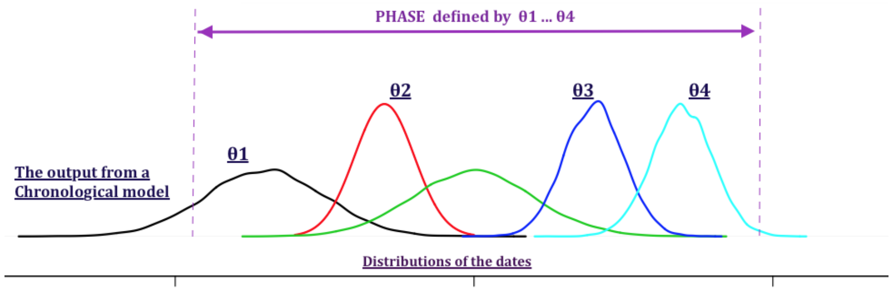
  This package cotains Statistical Tools for analysis the chronological modelling

Examples

1. Characterisation of a group of dates [begin / end /duration/ period]
2. Testing the presence of hiatus between two dates or two groups of dates. 1 of the target events

Phases : definition

A phase is a group of dates defined on the basis of objective criteria such as archaeological, geological or environmental criteria.



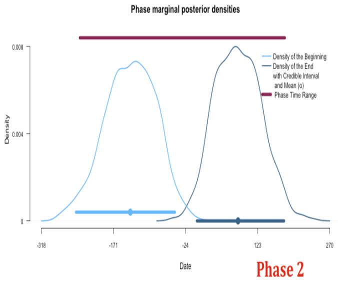
The collection of dates is estimated from a chronological model. [Chronomodel / Oxcal ...]

$$\text{Phase} = \{\theta_j,\ j \in J \subset \{1, ..., K\}\}$$

Estimation of the phase

$\text{Phase}_1 = \{\theta_j,\ j \in J \subset \{1, ..., K\}\}$.

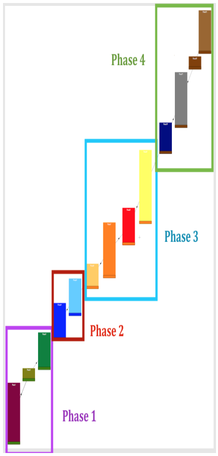
1. posterior distribution of the minimum $\alpha = \min_{j \in J} \theta_j$ $\rightsquigarrow$ Estimation of the beginning
2. posterior distribution of maximum $\beta = \max_{j \in J} \theta_j \rightsquigarrow$ Estimation of the end

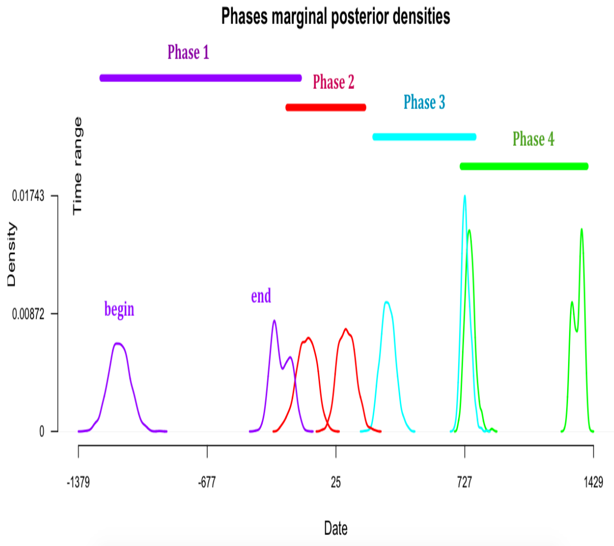


3. Phase time range The shortest interval that covers all the dates θ_j included in the phase at level 95%
   i.e. the shortest interval $[a, b] \subset T$ such that

$$P(\text{for all } j\ \theta_j \in [a, b] | M_1, ..., M_n) = P(a \leq \alpha \leq \beta \leq b | M_1, ..., M_n) = 95\%$$

Application to Volcanic eruptions [cont]

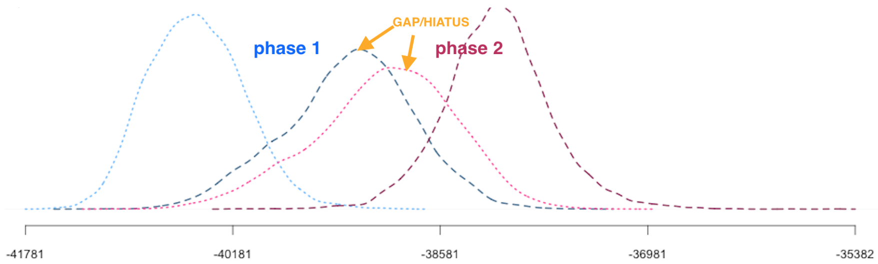




$P_1 = \{\theta_1, \theta_2, \theta_3\}, \cdots\cdots$

$P_4 = \{\theta_{10}, \theta_{11}, \theta_{12}, \theta_{13}\}$

Hiatus



Detection of a hiatus between two phases $\theta_j,\ j \in J_1$ and $\theta_j,\ j \in J_2$

1. $\beta_1 = \max_{j \in J_1} \theta_j$ and $\alpha_2 = \min_{j \in J_2} \theta_j$
2. Can we find $[c, d]$ such that

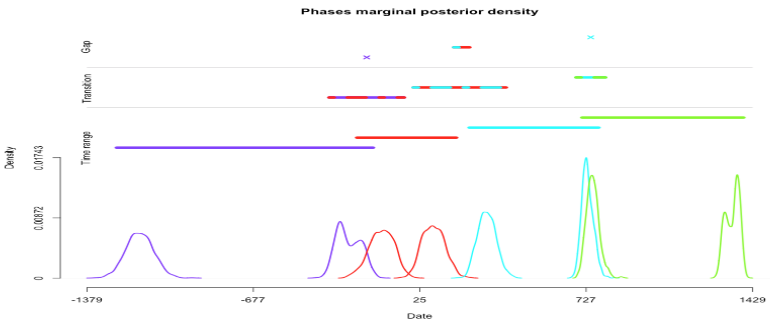
$$P(\beta_1 < c < d < \alpha_2 | M_1, ..., M_n) = 95\%?$$

Application cont.

Detection of hiatus :

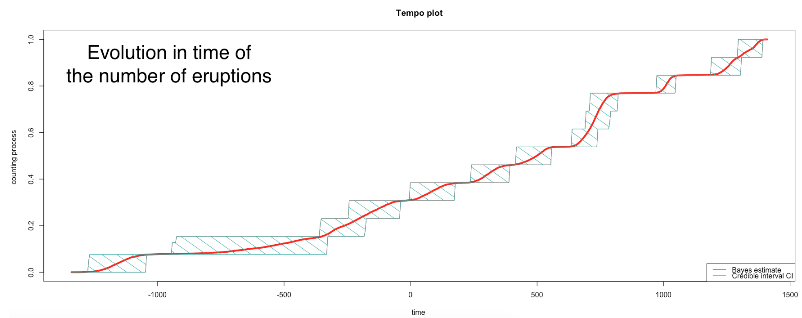
- A hiatus is detected between Phases 2 & 3. Estimation of the interval [170, 235]
- there is no gap between 1 & 2 and 3 & 4

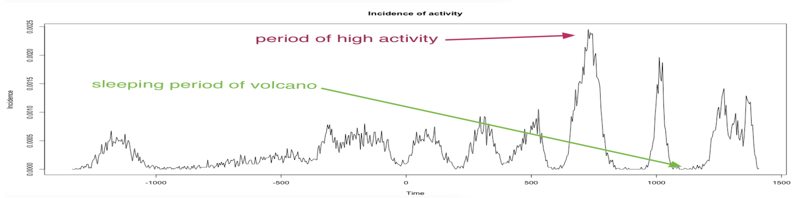
To summarise



Tempo plot Dye 2016

We evaluate the activity of volcano :





ArcheoPhases

Web application to use R functions available in RChronoModel package.