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Tools for Semiclassical Analysis

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The main goal of this course is to introduce the participants to modern tools for analyzing the Schrödinger equation in the transition régime between quantum and classical mechanics. In this régime the Planck constant $\hbar$ is small when it is compared to the macroscopic parameters (energy, masses, distances). The most accurate results proved in this field come from microlocal analysis: pseudodifferential and Fourier integral operator theory, WKB method. Here we shall explain these tools with emphasis on Gaussian coherent states because they give a nice intuition of the semiclassical transition. As an application we shall present a proof of the Gutzwiller trace formula.

Details:

1. Introduction to the time dependent and time independent Schrödinger equation.
2. Weyl calculus. Symbolic and functional calculus.
3. Properties of coherent coherent states. Time evolution by the Schrödinger equation.
4. The Gutzwiller trace formula.

Pre-requisites:
Functional analysis in Hilbert spaces. Fourier transform and Schwartz distribution theory.

Bibliography:

- D. Robert. *Autour de l'approximation semi-classique* Progress in math. Birkhaüser (1987)
- M. Dimassi, J. Sjöstrand. *Spectral asymptotics in the semi-classical limit* Lecture note series 268. Cambridge university press (1999)
- G.B. Folland. *Harmonic analysis in the phase space* Annals of math. studie, 122, Princeton university press (1989).
- M. Reed, B. Simon. *Methods of modern mathematical physics* I-IV, Academic Press, New York, (19975-78).