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Time Quasi-Periodic Solutions for Nonlinear Schroedinger Equations (NLSE)

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We present a new method, based on the recent works of J. Bourgain and myself, to construct time quasi-periodic solutions for NLSE. These NLSE are (infinite dimensional) Hamiltonian equations of motion. Quasi-periodic solutions are the equivalents of (finite dimensional) KAM tori. In spite of the analogy, our method does not utilize the inherent Hamiltonian structure. Instead it uses a Newton scheme directly on the equations.

This course is self-contained. No previous knowledge of KAM is needed. I start by introducing the central concept: multiscale analysis, which is crucial for the control of small divisors originating from the Newton scheme. I further illustrate and motivate this central concept on simple examples. I then explain our results, pointing out, whenever possible, what I believe are the specific aspects of the method, which enables us to treat systems closer to resonance.