A workshop celebrating Didier Robert

Abstracts

Víctor Arnaiz (Bordeaux) — *Two-microlocal properties of quantum limits of the Martinet sub-Laplacian.*

In this talk, we will describe the asymptotic properties of eigenfunctions of the Martinet sub-Laplacian in the high-energy limit. To this aim, we will decompose the phase-space in different regimes predicted by Rothschild-Stein estimates, and we will introduce two-microlocal semiclassical measures adapted to these regimes. Different normal form reductions will be needed to separate emerging effective dynamics ruled by harmonic or an-harmonic oscillators depending on the case. Some obstructions to dispersion will also be discused in terms of the eigenvalues of a family of Montgomery operators.

Jean-Marie Barbaroux (Toulon) — *Magnetic Dirac operators on domains: infinite mass boundary conditions versus zigzag boundary conditions.*

For two-dimensional Dirac operators defined on domains, with a perpendicular magnetic field, I will present some general spectral properties depending on the choice of boundary conditions. Then, through a detailed study of the energy dispersion curves in the half-plane case, I will highlight that the infinite-mass case generically captures the profile of these curves for all boundary conditions that interpolate between infinite-mass and zig-zag boundary conditions, but that the pure zig-zag case reveals a different behaviour. I will eventually show that this has an impact on the bulk-edge conductance for the system.

Monique Dauge (Rennes) — *Old and new about corners and pseudo-corners.*

As a remembrance of my old times in Nantes, I will briefly recall the lack of regularity for solutions of elliptic boundary value problems near corners, addressing the Laplace equation with Dirichlet conditions as an example. Then I will address a more paradoxical situation in which families of domains are defined, depending continuously of a small parameter ε, and so that in the limit $\varepsilon \to 0$ the regularity of the boundary drops, creating a sort of "pseudo-corner". We will see that, though being finite for each positive ε, some Sobolev norms of solutions blow up in a quantified way as $\varepsilon \to 0$. These new results are based upon the new paper [Costabel, Dalla Riva, Dauge, Musolino; JDE 2025] in which convergence results are proved.

Stephan De Bièvre (Lille)— *Quasi-probabilities at the service of quantum mechanics.*

This year we are celebrating the 80th anniversary of Didier and the 100th anniversary of quantum mechanics! Over all these years, quantum mechanics has inspired beautiful quantum and semi-classical mathematics, from Didier and from many others. The quantum world has nevertheless not revealed all its secrets and one question in particular is still relevant: "But how is quantum mechanics so different from classical mechanics?" In this presentation I will illustrate one aspect of this question: the use of quasi-probabilities (i.e. complex measurements of mass 1 such as the Wigner function) to (attempt to) capture the classical-quantum boundary.

Clotilde Fermanian Kammerer (Angers) — *Spring variations on wave packets.*

In this talk, revisiting the early properties of wave packets, we will present different aspects of wave packets. We will emphasize how one can extend this notion on nilpotent stratified Lie groups, replacing the Fourier transform in the Euclidean setting by the Fourier transform constricted on the group via representation theory. We will discuss how these constructions, inspired by the works of Didier Robert, allow to exhibit specific dynamics of subelliptic operators.

Patrick Gérard (Paris Saclay) — *The Benjamin-Ono equation : explicit formula and solitons.*

I will consider the Benjamin-Ono equation, which is a dispersive equation on the line and admits a Lax pair structure involving operators on the Hardy space. Using this structure, I will derive an explicit formula for all the solutions, and a new proof of the classification of soliton solutions for this equation.

Bernard Helffer (Nantes) — *Une promenade dans l'oeuvre scientifique de Didier Robert.*

Luc Hillairet (Orléans)— *Approximation of singular Laplace operators by regular ones.*

We define a systematic way of regularising a sub-Riemannian structure by a sequence of Riemannian manifolds and we study the associated Laplace operators. We also apply this method to other types of singular operators.

Caroline Lasser (Munich) — *Variational Gaussian wave packets and magnetic Schrödinger dynamics.*

The talk considers the semiclassical magnetic Schrödinger equation, which describes the dynamics of charged particles under the influence of an electro-magnetic field. We analyse the accuracy of Gaussian wave packet approximation obtained via the time-dependent Dirac-Frenkel variational principle, emphasizing the differences to the classical approach as developed, e.g., in the monograph by Combescure and Robert. If time permits, we address the numerical time integration of the equations of motion, before presenting numerical experiments including the dynamics of an electronic wave packet within a Penning trap.

Alberto Maspero (SISSA) — *One dimensional energy cascades in a fractional quasilinear NLS.*

We consider the problem of transfer of energy to high frequencies in a quasilinear Schrödinger equation with sublinear dispersion, on the one dimensional torus. We exhibit initial data undergoing finite but arbitrary large Sobolev norm explosion: their initial norm is arbitrary small in Sobolev spaces of high regularity, but at a later time becomes arbitrary large. We develop a novel mechanism producing instability, based on Mourre's positive commutator estimates. This is a joint work with F. Murgante, but inspired by conversations that I had with Didier almost 10 years ago.

Julien Royer (Toulouse) — *Energy decay for the damped wave equation.*

We consider the question of energy decay for the (possibly damped) wave equation in the asymptotically Euclidean setting. In even dimensions, we prove optimal estimates and provide the asymptotic profile of the solution for large times. In odd dimensions, we improve the best known estimates, and in particular we go beyond the decay rate which is optimal in even dimensions. The proof is based on resolvent estimates for the corresponding Helmholtz equation near energy 0. This is a joint work with Rayan Fahs.

Laurent Thomann (Nancy) — *Wellposedness for the Φ^4_3 model with harmonic oscillator.*

We prove the local wellposedness of the (renormalized) Φ^4_3 model associated with the harmonic oscillator on $\mathbb{R}^3$, that is, the equation formally written as

$$\partial_t X + HX = -X^3 + \infty \cdot X + \xi, \quad t > 0, \quad x \in \mathbb{R}^3,$$

where $H := -\Delta_{\mathbb{R}^3} + |x|^2$ and ξ denotes a space-time white noise. Using para- differential techniques developed by Mourrat and Weber for an analogous problem but with periodic conditions and pseudo-differential calculus to study the interactions due to the harmonic oscillator, we show that this renormalized problem is locally well-posed. This is a joint work with Aurélien Deya (Nancy) and Reika Fukuizumi (Tokyo).

San Vũ Ngọc (Rennes) — *Semiclassical analysis and Fourier integral operators for Holography.*

In this talk I will report on a collaboration with the holographic team of the B-Com technological research institute. Motivated by very applied challenges about digital hologram processing, we realized that semiclassical analysis as developed by Didier Robert and others, and in particular the theory of coherent states and Fourier integral operators, gives a new "phase space" approach to these questions, which can produce both practical and theoretical results. In a joint work with Patrick Gioia, using the light model given by the scalar Helmholtz equation, we describe the canonical (ie symplectic) transformations associated with holographic transformations and prove an accurate (non-unitary) Egorov theorem.

Xue Ping Wang (Nantes) — *The Kramers-Fokker-Planck equation with a decaying potential.*

In this talk, we report some old and new results for the Kramers-Fokker-Planck equation with a decaying potential, obtained by methods from quantum scattering. This includes the large-time behavior of solution, the limiting absorption principles and the exponential decay of eigenfunctions.