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Rigidité d'Einstein du plan hyperbolique complexe. (French. English, French summaries) [Einstein rigidity of the complex hyperbolic plane]

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Using a scalar curvature estimate derived from Seiberg-Witten theory, C. LeBrun [see Math. Res. Lett. **2** (1995), no. 1, 1–8; MR1312972 (95m:53067)] proved that smooth compact quotients of the complex hyperbolic plane carry a unique Einstein metric up to diffeomorphism and rescaling.

The paper under review extends this result to the infinite volume setting. It is shown that every Einstein metric on $B^4 \subset \mathbb{C}^2$ asymptotic to the Bergman metric is equal to it, up to a diffeomorphism. The proof relies on the construction of a solution of the Seiberg-Witten equations with a strong exponential decay property towards the boundary. It is proved that such a solution exists on any 4-dimensional manifold which has a CR-boundary at infinity, an adapted Spin^c structure with a nonzero Kronheimer-Mrowka invariant, and an asymptotically complex hyperbolic Einstein metric.

Reviewed by *Tedi C. Draghici*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.