

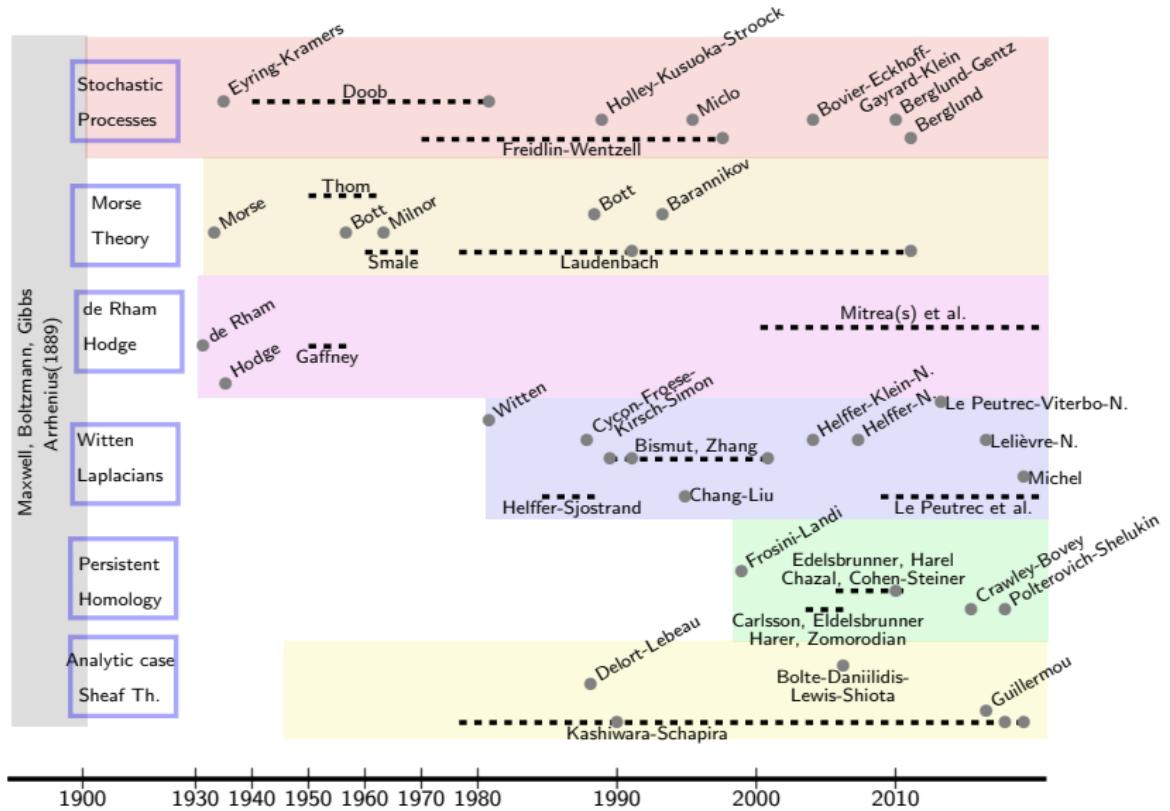
Persistence
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Arrhenius
law (Part
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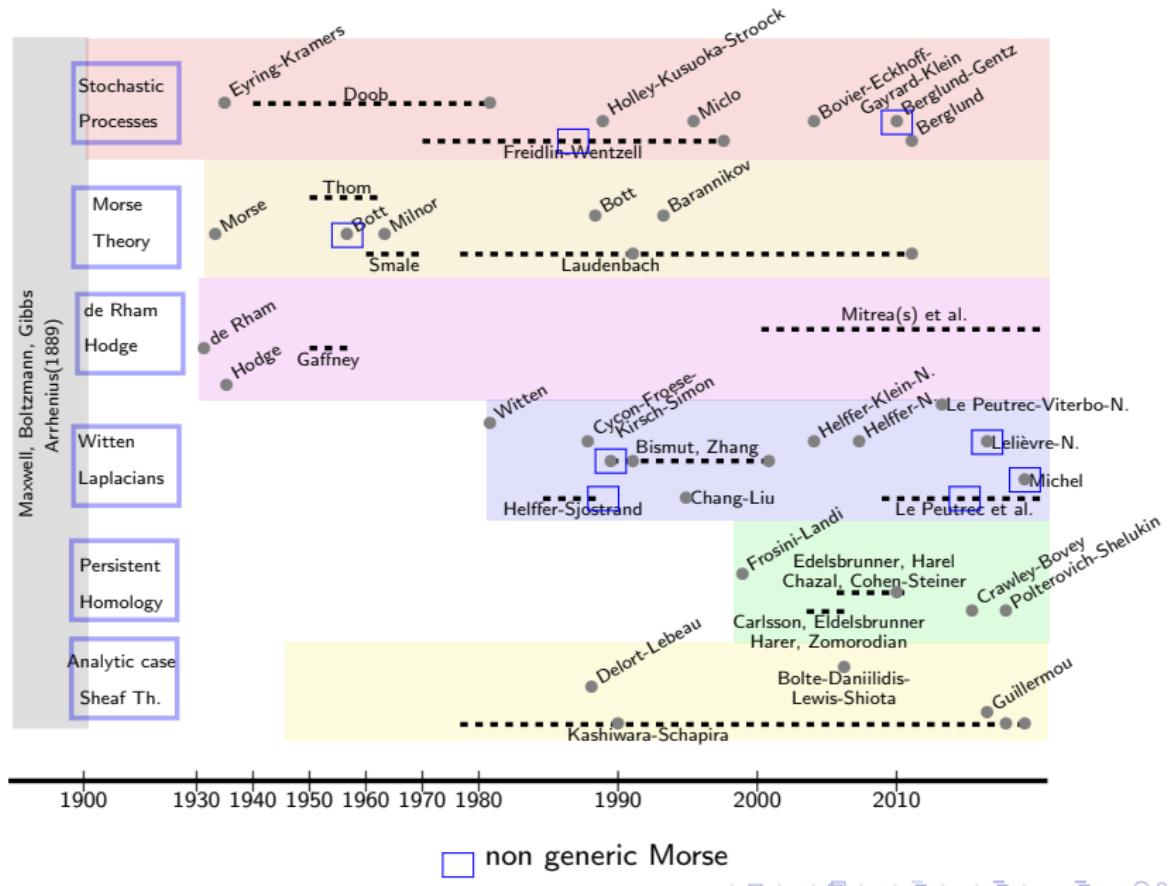
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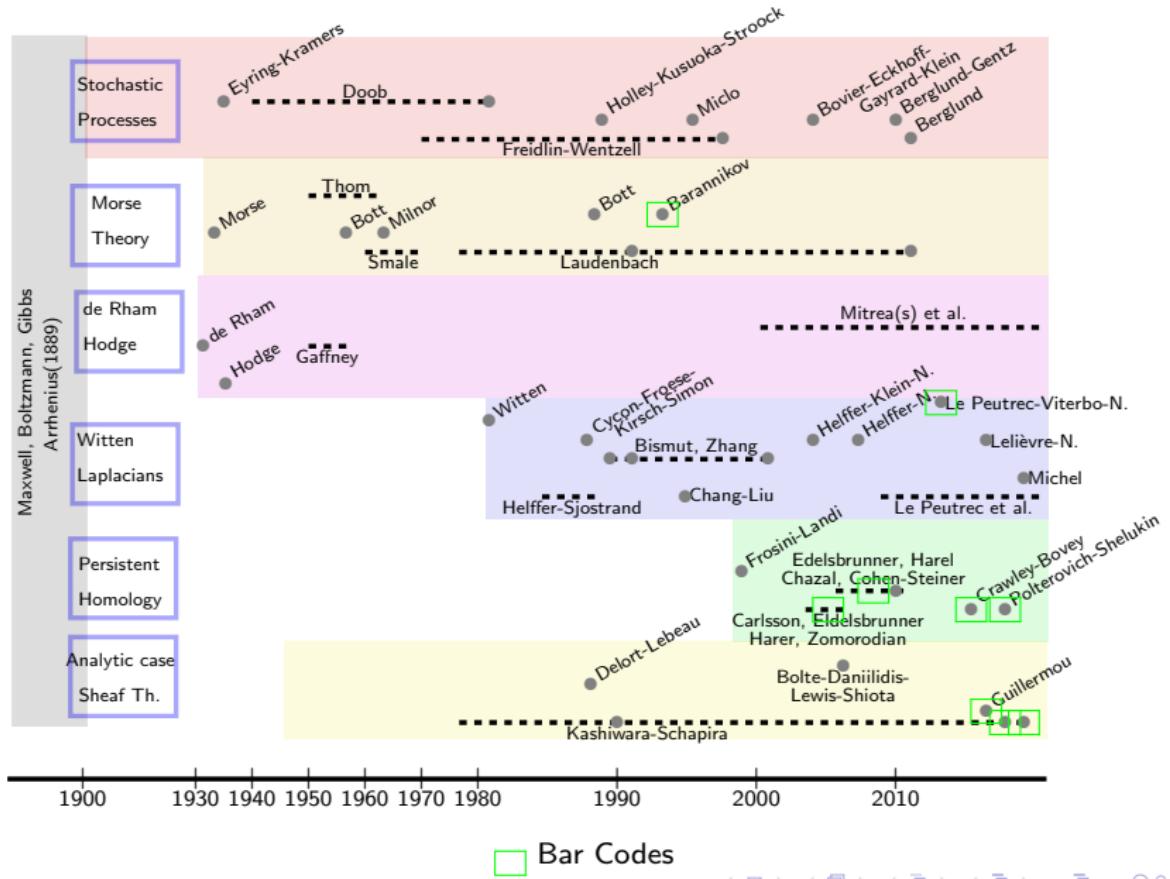
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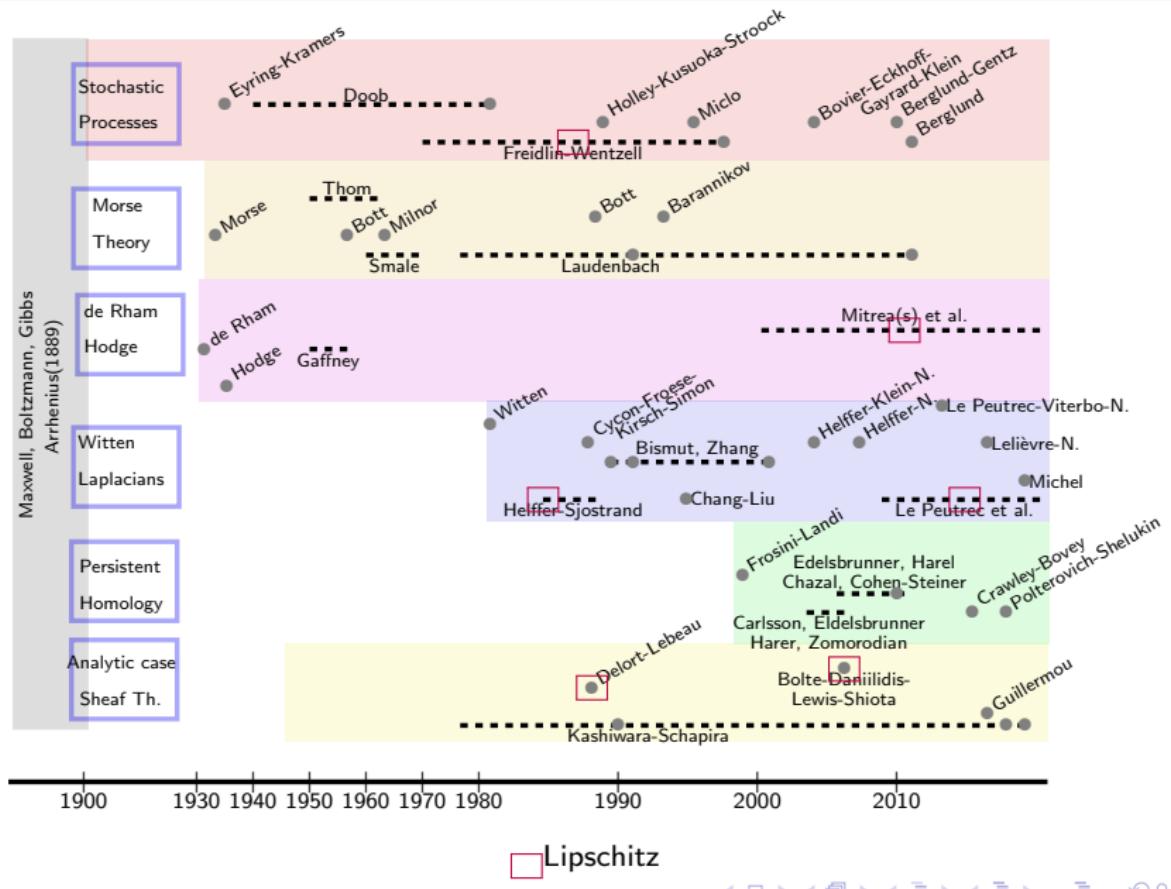
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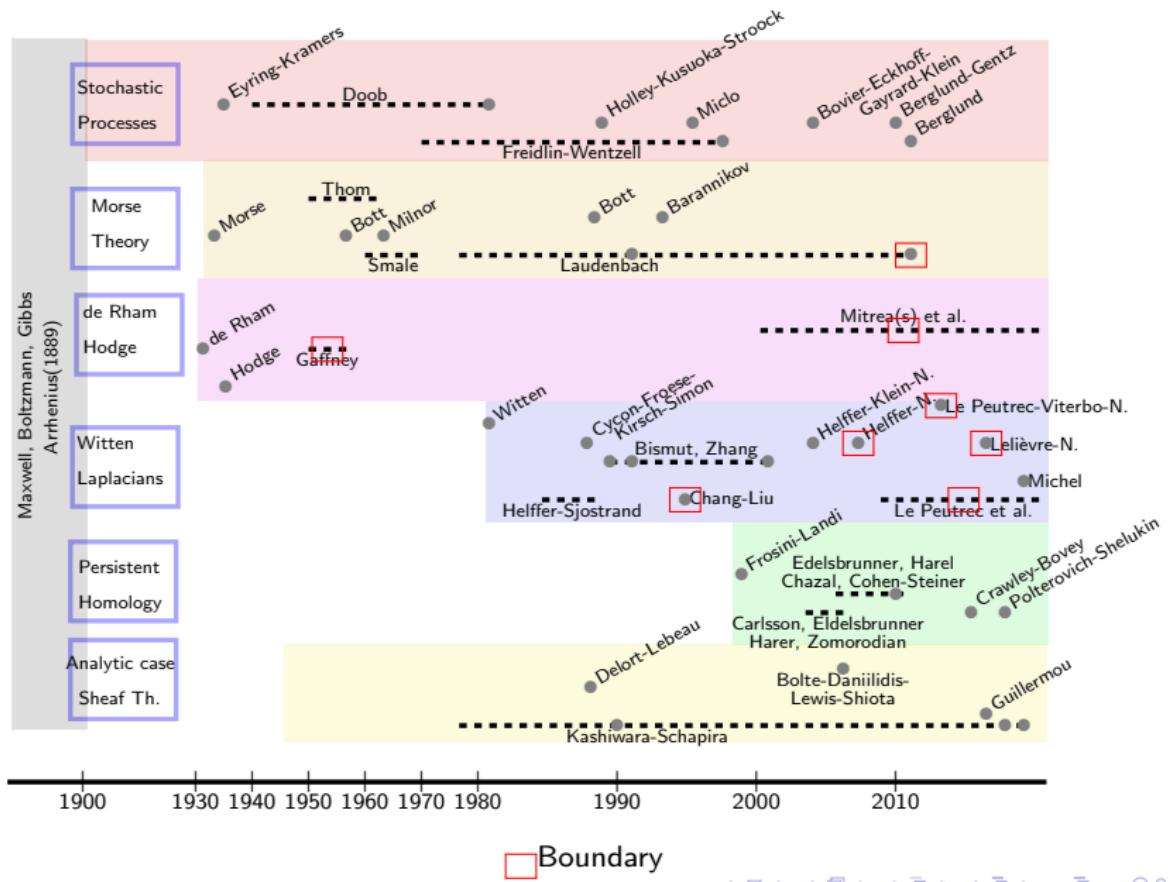
Nantes, 26/04/2019

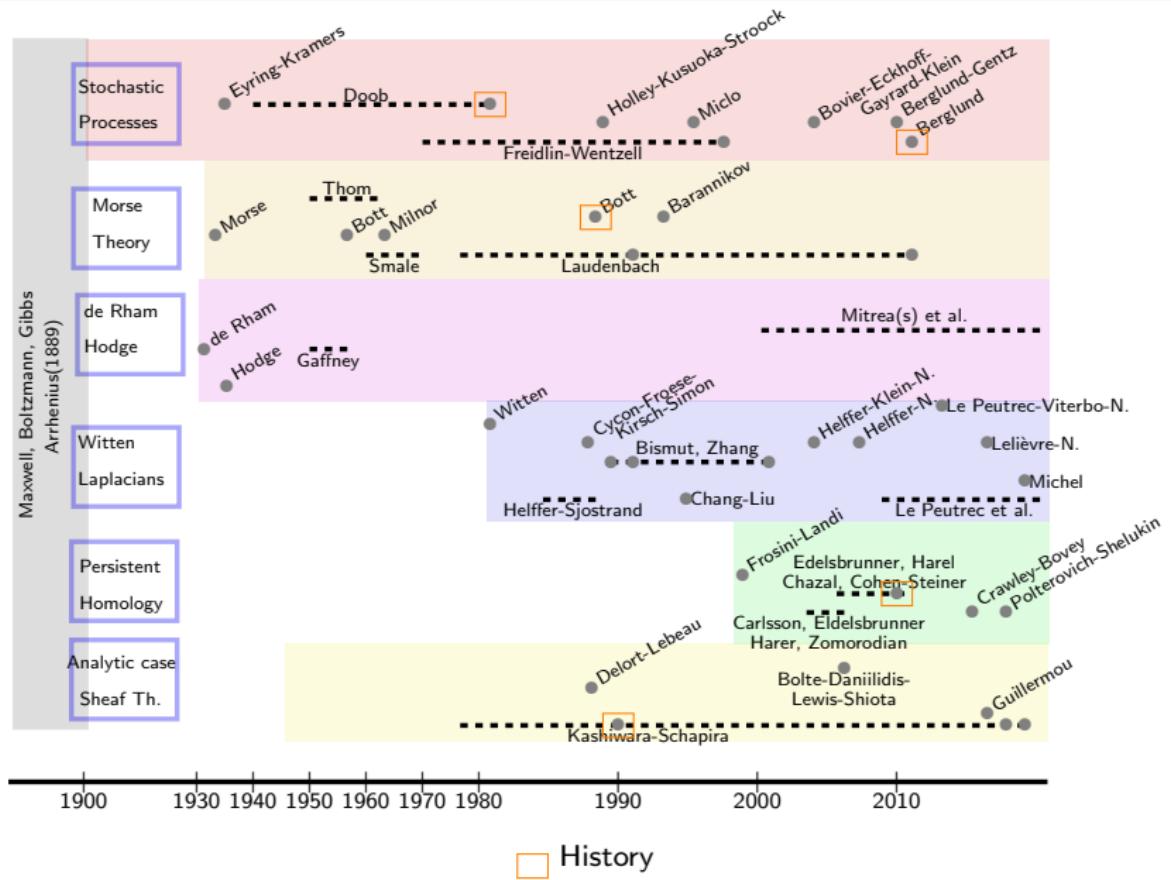












Bar codes

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ogy and
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Assumption

- (M, g) is a closed (compact) riemannian manifold.
- f Lipschitz.
- $f \in C^\infty$ and $df \neq 0$ in $f^{-1}(\mathbb{R} \setminus \{c_1, \dots, c_{N_f}\})$.
 c_1, \dots, c_{N_f} : "critical values".

Alternatively, M is real analytic (compact) and f is Lipschitz and subanalytic.

→ not yet

Definition

A bar code associated to f is a finite family $\mathcal{B}_f = (\underbrace{[a_\alpha^{(p)}, b_\alpha^{(p+1)}]}_{\text{degree } p})_{\alpha \in A}$ such that

$a_\alpha \in \{c_1, \dots, c_{N_f}\}$, $b_\alpha \in \{c_2, \dots, c_{N_f}, +\infty\}$ and

$$H^{(p)}(f^b, f^a; \mathbb{K}) \sim \bigoplus_{a_\alpha^{(p-1)} < a < b_\alpha^{(p)} < b} \mathbb{K} b_\alpha^{(p)} \bigoplus \bigoplus_{a < a_\alpha^{(p)} < b < b_\alpha^{(p+1)}} \mathbb{K} a_\alpha^{(p)}.$$

\mathcal{B}_f unique modulo permutation and the addition of empty bars.

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Stability theorem

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Definition

Bottleneck distance $\mathcal{B} = ([a_\alpha, b_\alpha])_{\alpha \in A}$, $\mathcal{B}' = ([a'_\alpha, b'_\alpha])_{\alpha \in A}$.

$$d_{bot}(\mathcal{B}, \mathcal{B}') = \min_{\sigma \in \mathfrak{S}(A)} \max_{\alpha \in A} \max(|a_\alpha - a'_{\sigma(\alpha)}|, 1_{\mathbb{R}}(\min(b_\alpha, b'_{\sigma(\alpha)}))|b_\alpha - b'_{\sigma(\alpha)}|).$$

The same A is obtained after possibly adding empty bars.

Theorem (Cohen-Steiner–Edelsbrunner–Harrel (07), Kashiwara–Schapira (16))

For two functions f, g which satisfy our Assumption,

$$d_{bot}(\mathcal{B}_f, \mathcal{B}_g) \leq \|f - g\|_{C^0}$$

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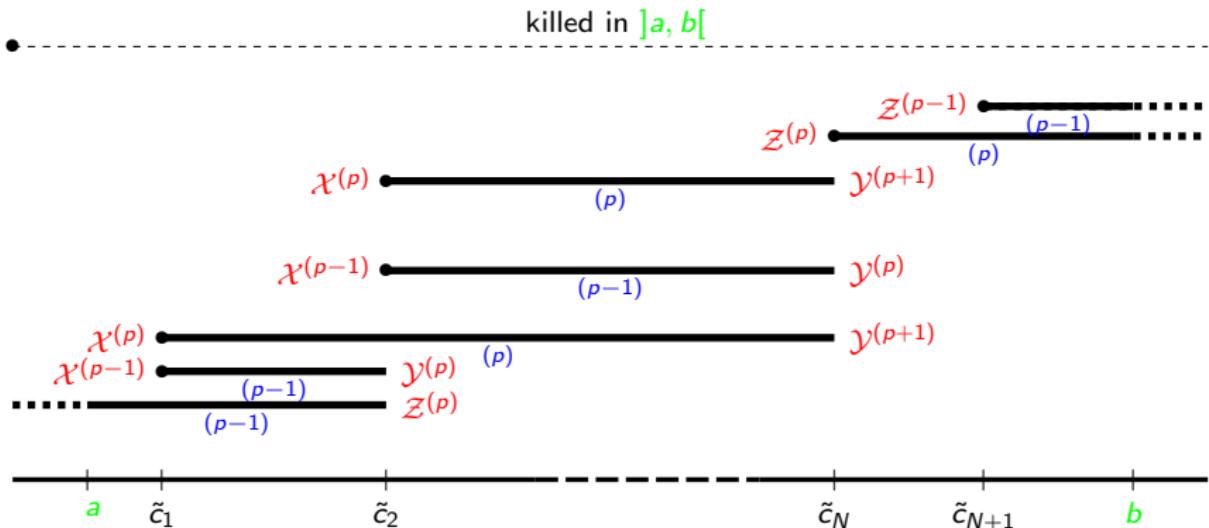
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Some notations in $f^{-1}([a, b])$, $a < b$ non “critical”

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Labelling of the endpoints: $\mathcal{J}(a, b) = \mathcal{X}(a, b) \sqcup \mathcal{Y}(a, b) \sqcup \mathcal{Z}(a, b)$



$$\mathcal{X}^* = \mathcal{X}^*(a, b) \text{ (lower)}, \mathcal{Y}^* = \mathcal{Y}^*(a, b) \text{ (upper)}, \mathcal{Z}^* = \mathcal{Z}^*(a, b) \text{ (lonely)}$$

$$\beta^{(p)}(f^b, f^a; \mathbb{K}) = \dim H^{(p)}(f^b, f^a; \mathbb{K}) = \#\mathcal{Z}^{(p)}(a, b)$$

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Definition

Differential operators

$$d_{f,h} = e^{-\frac{f}{h}}(hd)e^{\frac{f}{h}} = hd + df \wedge , \quad d_{f,h}^* = e^{\frac{f}{h}}(hd^*)e^{-\frac{f}{h}} = hd^* + i_{\nabla f}$$

$$\Delta_{f,h} = (d_{f,h} + d_{f,h}^*)^2 = d_{f,h}^* d_{f,h} + d_{f,h} d_{f,h}^*.$$

Closed operators in $L^2(f_a^b, \Lambda T^* M)$

$$D(d_{f,f^{-1}([a,b]),h}) = \{\omega \in L^2, d_{f,h}\omega \in L^2, t\omega|_{f=a} = 0\} .$$

$$D(d_{f,f^{-1}([a,b]),h}^*) = \{\omega \in L^2, d_{f,h}^*\omega \in L^2, n\omega|_{f=b} = 0\} .$$

$$\left[\text{conseq. } D(d_{f,f^{-1}([a,b]),h}) \cap D(d_{f,f^{-1}([a,b]),h}^*) \stackrel{\text{Gaffney}}{\subset} W^{1,2}(f^{-1}([a,b]), \Lambda T^* M) . \right]$$

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$$d_{f,f^{-1}([a,b]),h}(z - \Delta_{f,f^{-1}([a,b],h)})^{-1} = (z - \Delta_{f,f^{-1}([a,b],h)})^{-1} d_{f,f^{-1}([a,b]),h} \cdot \\ F_{[0,\alpha],[a,b],h}^{(p)} = \text{Ran } 1_{[0,\alpha]}(\Delta_{f,f^{-1}([a,b],h)}^{(p)}) \quad , \quad \delta_{[0,\alpha],[a,b],h} = d_{f,f^{-1}([a,b],h)} \Big|_{F_{[0,\alpha],[a,b],h}}$$

$$\delta_{[0,\alpha],[a,b],h}^{(p)} : F_{[0,\alpha],[a,b],h}^{(p)} \rightarrow F_{[0,\alpha],[a,b],h}^{(p+1)}$$

$$\Delta_{f,f^{-1}([a,b]),h} = \underbrace{\delta_{[0,\alpha],[a,b],h}^* \delta_{[0,\alpha],[a,b],h}}_{A+B} + \underbrace{\delta_{[0,\alpha],[a,b],h} \delta_{[0,\alpha],[a,b],h}^*}_{AB=BA=0}.$$

$$F_{[0,\alpha],[a,b],h} = \underbrace{\text{Ran } \delta_{[0,\alpha],[a,b],h}}_{\ker \delta_{[0,\alpha],[a,b],h}} \overset{\perp}{\oplus} \underbrace{\ker(\Delta_{[0,\alpha],[a,b],h})}_{\ker \delta_{[0,\alpha],[a,b],h}} \overset{\perp}{\oplus} \text{Ran } \delta_{[0,\alpha],[a,b],h}^*$$

Consequences:

- a) $\ker(\Delta_{f,f^{-1}([a,b],h)}^{(p)}) = \ker(\delta_{[0,\alpha],[a,b],h}^{(p)}) / \text{Ran } \delta_{[0,\alpha],[a,b],h}^{(p-1)} \sim \ker(d_{0,f^{-1}([a,b],h)}^{(p)}) / \text{Ran } d_{0,f^{-1}([a,b],h)}^{(p)} \sim H^{(p)}(f^b, f^a; \mathbb{R})$
- b) Eigenvalues of $\Delta_{f,f^{-1}([a,b]),h}$ lying in $]0, \alpha]$ are the square of singular values of $\delta_{[0,\alpha],[a,b],h}$ (counted twice).

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- b) Eigenvalues of $\Delta_{f,f^{-1}([a,b]),h}$ lying in $]0, \alpha]$ are the square of singular values of $\delta_{[0,\alpha],[a,b],h}$ (counted twice).

$D(d_{f,f^{-1}([a,b]),h}) \subset D(d_{g,f^{-1}([a',b]),h})$ for any Lipschitz g and $a' \leq a$.

Main result

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Theorem

$$\{c_1, \dots, c_{N_f}\} \cap [a, b] = \{\tilde{c}_1, \dots, \tilde{c}_N\}. \quad 0 < \delta_1, \delta_2 < \min \frac{c_n - c_{n-1}}{16}.$$

$$F_{[0, \tilde{o}(1)], [a, b], h}^{(p)} = \text{Ran } 1_{[0, e^{-\frac{\varepsilon}{h}}]}(\Delta_{f, f^{-1}([a, b]), h}^{(p)}) \quad (\exists \varepsilon > 0 \text{ indep. of } h).$$

The singular values of $d_{f, f^{-1}([a, b]), h}|_{F_{[0, \tilde{o}(1)], [a, b], h}^{(p)}}$, are $(\mu_j^{(p), h})_{j \in \mathcal{J}^{(p)}(a, b)}$,

$$\lim_{h \rightarrow 0} -h \log \mu_j^{(p), h} = b_\alpha^{(p+1)} - a_\alpha^{(p)} \quad \text{if } j = (\alpha, a_\alpha^{(p)}) \in \mathcal{X}^{(p)}(a, b),$$

$$\mu_j^{(p), h} = 0 \quad \text{if } j \in \mathcal{Y}^{(p)}(a, b) \sqcup \mathcal{Z}^{(p)}(a, b).$$

There exists a δ_1 -family of quasimodes $(\varphi_j^h)_{j \in \mathcal{J}(a, b)}$ which is

$\tilde{O}(e^{-\frac{\delta_1}{h}})$ -orthonormal and such that $\mathcal{V}^{(p), h} = \text{Vect}(\varphi_j^h, j \in \mathcal{J}^{(p)}(a, b))$ satisfies

$$\vec{d}(\mathcal{V}^{(p), h}; F_{[0, \tilde{o}(1)], [a, b], h}^{(p)}) + \vec{d}(F_{[0, \tilde{o}(1)], [a, b], h}^{(p)}; \mathcal{V}^{(p), h}) = \tilde{O}(e^{-\frac{\delta_1}{h}}).$$

$$\begin{array}{ccc} \mathcal{V}^{(p), h} & \xrightarrow{d_{f, f^{-1}([a, b]), h}^{(p)}, T_{\delta_2}} & L^2(f^{-1}([a, b])) \\ & \searrow & \uparrow C^h \\ & \Pi_{F_{[0, \tilde{o}(1)], [a, b], h}^{(p)}} d_{f, f^{-1}([a, b]), h}^{(p)}, T_{\delta_2} & F_{[0, \tilde{o}(1)], [a, b], h}^{(p)} \end{array}$$

$$\|C^h\| = \tilde{O}(e^{\frac{2\delta_2}{h}}).$$



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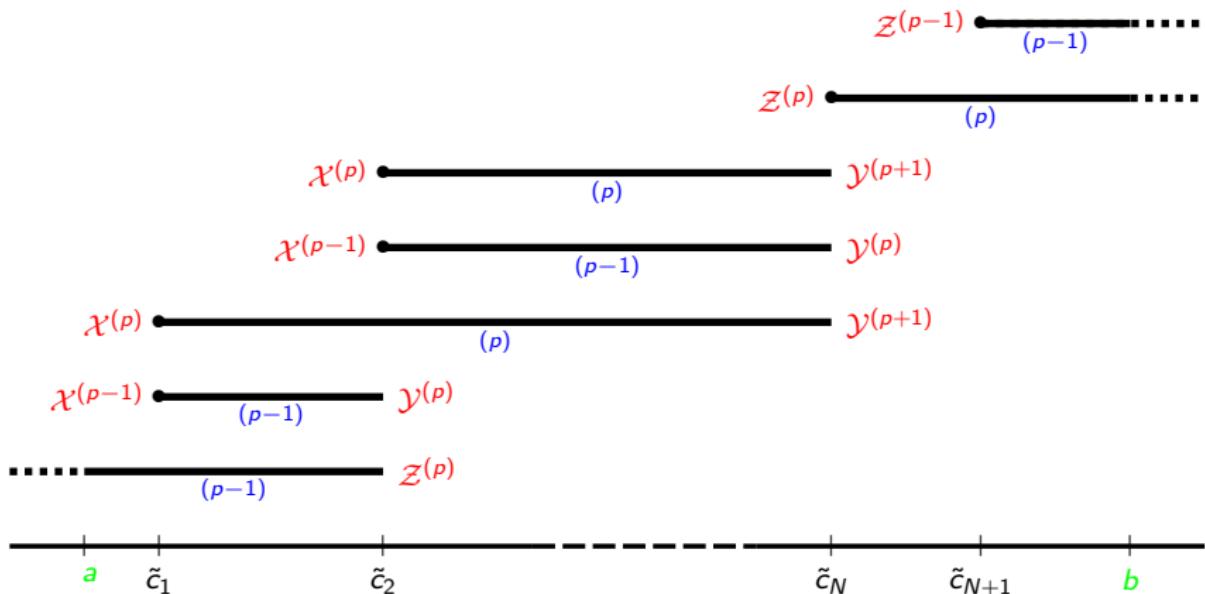
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Explanation

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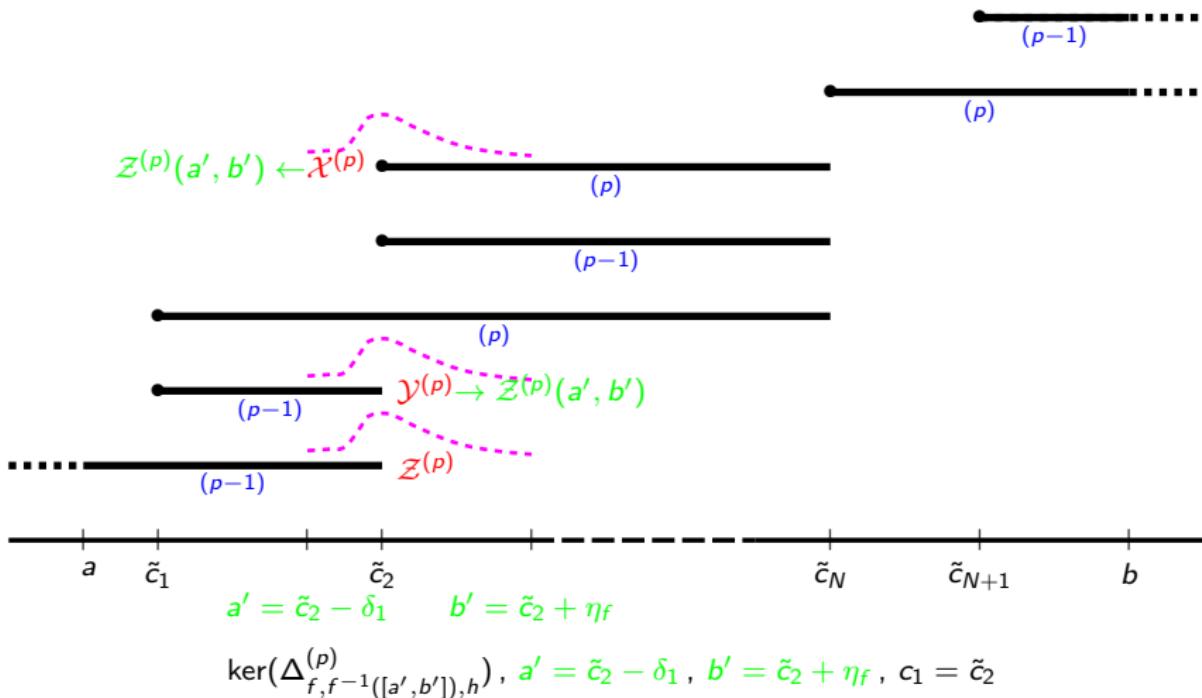
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$$\mathcal{X}^* = \mathcal{X}^*(a, b) \text{ (lower)}, \mathcal{Y}^* = \mathcal{Y}^*(a, b) \text{ (upper)}, \mathcal{Z}^* = \mathcal{Z}^*(a, b) \text{ (lonely)}$$

Explanation

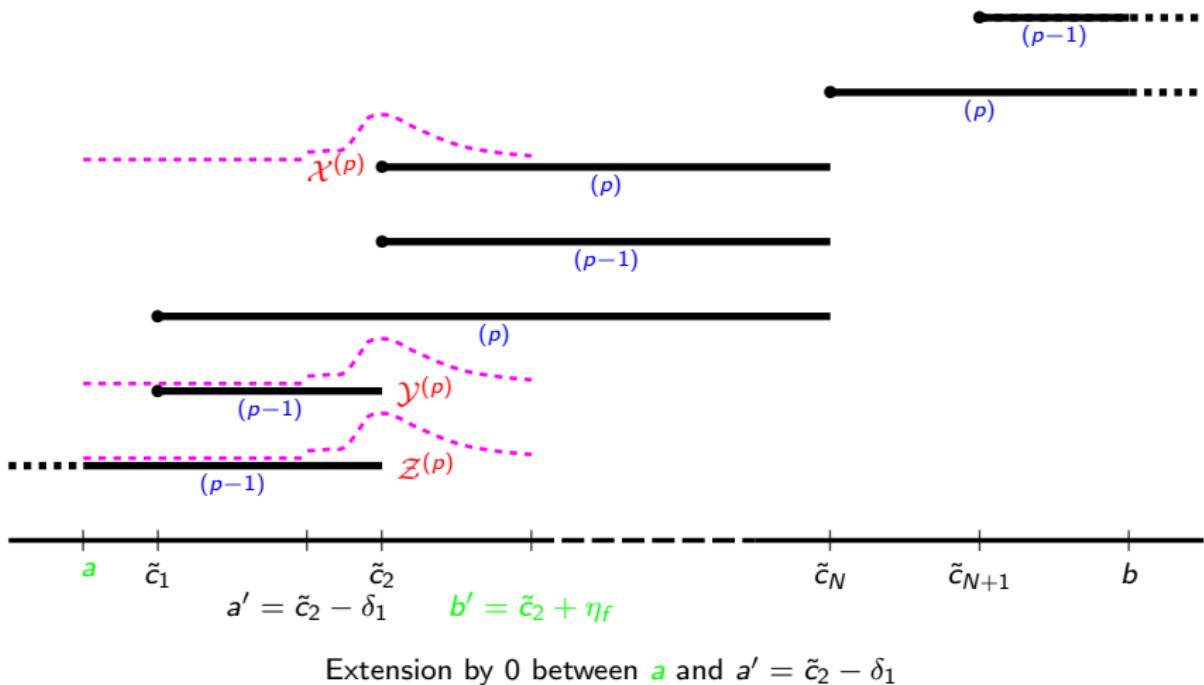
Persistence cohomol- ogy and Arrhenius law (Part I),



Explanation

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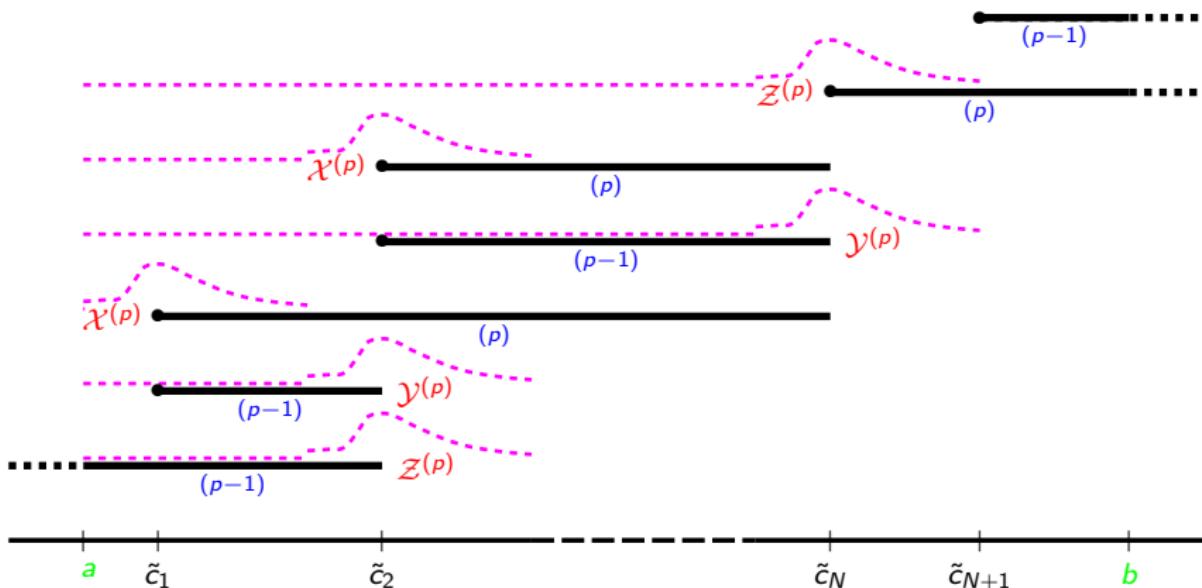
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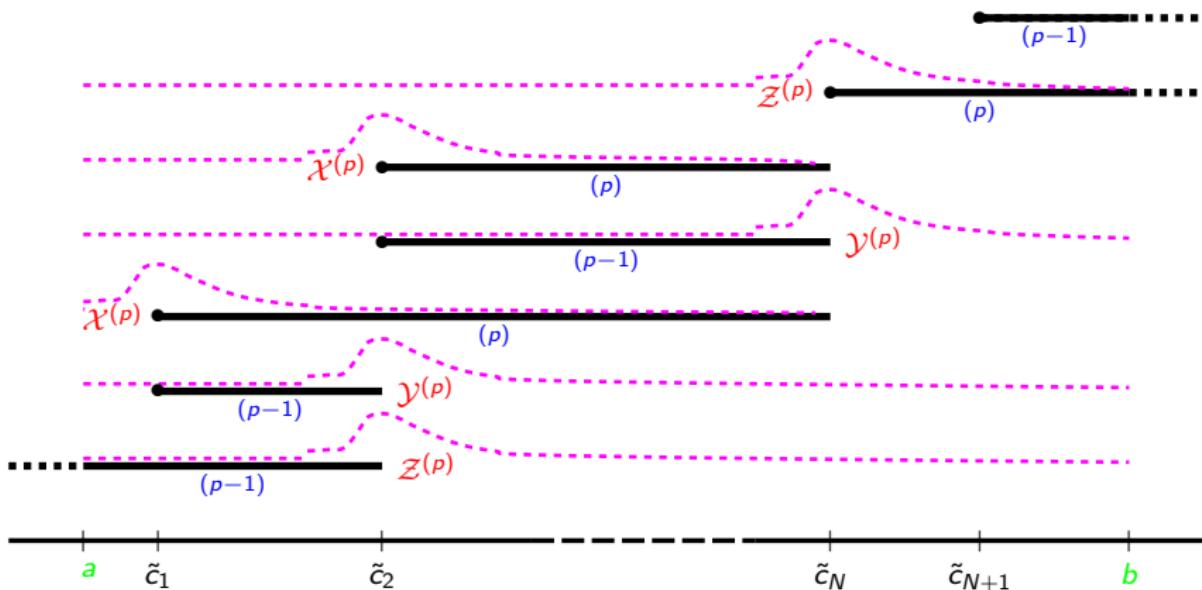


Same local construction for all \tilde{c}_n in $]a, b[$

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Global construction.

The truncation operator T_{δ_2} truncates just before the upper end $\in \mathcal{Y}^{(p+1)}$ of the bar.

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Happy birthday Bernard!