Semaine sur les systèmes dynamiques : la théorie d'Aubry - Mather du 8 au 12 Avril 2013

Pendant cette semaine le Laboratoire de Mathématique Jean Leray accueillira Marie-Claude Arnaud (Université d'Avignon), Alfonso Sorrentino (Università degli Studi Roma Tre) et Daniel Massart (Université de Montpellier).

Marie-Claude Arnaud est invitée au Colloquium le jeudi 11 Avril et au séminaire d'analyse le vendredi 12 Avril.
Alfonso Sorrentino fera 3 exposés sur "The principle of least action in Hamiltonian dynamics".

PROGRAMME

Mardi 9 Avril (salle de séminaires)

14:00-14:50 Alfonso Sorrentino, The principle of least action in Hamiltonian dynamics I
15:00-16:50 Alfonso Sorrentino, The principle of least action in Hamiltonian dynamics II

Jeudi 11 Avril demi-journée: La théorie d'Aubry-Mather et isospectralité (salle de séminaires)

09:30 - 10:20 Alfonso Sorrentino, The principle of least action in Hamiltonian dynamics, III
10:30 - 11:20 Daniel Massart, Differentiability of Mather's β-function on the two-torus
11:30 - 12:20 Georgi Popov, Isospectral deformations and Mather's α-function

Jeudi 11 Avril, 17:00-18:00 Colloquium

Marie-Claude Arnaud, Étude dynamique des twists conservatifs : courbes invariantes et zones d' instabilités (salle de séminaires)

Vendredi 12 Avril, Séminaire Analyse

14:00-15:00 Marie-Claude Arnaud, Hamiltoniens de Tonelli sans point conjugués (salle Eole)

RÉSUMÉ

Hamiltoniens de Tonelli sans point conjugués.
Marie-Claude Arnaud

Résumé: Burago et Ivanov on démontré il a a une quinzaine d'année la conjecture de Hopf suivante: une métrique du tore sans point conjugués est plate. Se pose alors la question pour les hamiltoniens de Tonelli, qui sont des généralisations des métriques riemanniennes. On verra qu'alors l'espace des phases est feuilleté en tores Lipschitz lagrangiens invariants par le flot hamiltonien, et que la dynamique est d'entropie topologique nulle.

The principle of least action in Hamiltonian dynamics.
Alfonso Sorrentino

In these three lectures I shall present John Mather's variational approach to the study of convex and superlinear Hamiltonian systems, what is generally called Aubry-Mather theory. Starting from the crucial observation that invariant Lagrangian graphs can be characterised in terms of their "action-minimizing properties", we shall investigate how analogue features can be traced in a more general setting, namely the so-called Tonelli Hamiltonian systems. This different point of view will bring to light a plethora of compact invariant subsets of the system that, under many points of view, could be considered as generalisation of invariant Lagrangian graphs, despite not being in general either submanifolds or regular. We shall discuss their structure and their symplectic properties, as well as their relation to the dynamics of the system. Moreover, I shall point out some connections of this theory to other topics, such as classical mechanics, Hamilton-Jacobi equation (weak KAM theory), symplectic geometry, Hofer's geometry etc...

Differentiability of Mather's β-function on the two-torus.
Daniel Massart

Let L be a Tonelli Lagrangian $T\mathbb{T}^2 \times \mathbb{T} \longrightarrow \mathbb{R}$. To any Borel probability measure μ on $T\mathbb{T}^2 \times \mathbb{T}$, such that μ is invariant under the Euler-Lagrange flow of L, we may associate a homology class, or average rotation number, $[\mu]$. Mather's β-function maps a homology class to the minimal action (with respect to L) of a measure in this homology class :

$$\begin{aligned} \beta : H_1(\mathbb{T}^2, \mathbb{R}) &\longrightarrow \mathbb{R} \\ h &\longmapsto \min \textstyle\int L d\mu \; : \; [\mu] = h. \end{aligned}$$

Then we ask the question: is it true that β is always differentiable at a completely irrational homology class ? In this talk I shall explain the affirmative answer to this question, in the particular case when the configuration space is the two-torus.

Isospectral deformations, Mather's α-function and spectral rigidity
Georgi Popov

Consider a family of Laplace-Beltrami operators corresponding to a smooth deformation of Riemannian metrics on a compact manifold with or without boundary. Suppose that the initial metric is either completely integrable or close to a non-degenerate completely integrable metric (KAM system). If the deformation is isospectral we prove that the values of the corresponding Mather's α-function given by the average action on the KAM tori is constant along the deformation. As an application we obtain infinitesimal rigidity of Liouville billiard tables. The proof is based on a construction of quasi-modes associated with KAM tori. The results are based on a joint work with Peter Topalov (Northeastern University).