PhD position in Applied Mathematics

Randomized methods for model order reduction of parameter-dependent evolution equations

Joint PhD program between

Centrale Nantes (France) and Stevens Institute of Technology (Hoboken, USA)

Project description

Various tasks in optimization, control, data assimilation or uncertainty quantification for complex systems require the numerical solution of parameter-dependent equations for many instances of the parameters. Examples for the latter are material properties or geometric parameters that describe the shape of an object. This leads to prohibitive computational costs unless the complexity of the numerical models has been appropriately reduced. For instance, in order to monitor the seismicity in a certain area with existing techniques, just one single simulation lasts several hours on a high-end computer cluster and running the very many simulations that are needed for monitoring would last several years. Model order reduction (MOR) methods can bring down the computational time for one simulation to the order of minutes, thus making the development of monitoring and prediction tools possible in the first place.

Over the past years, the design of MOR methods has become an active field of research. These methods include low-rank approximation or reduced basis methods which are able to significantly speed up the computations without jeopardizing the accuracy. The key idea is to build a reduced approximation from solutions of the problem for certain parameters and then apply e.g., a singular value decomposition to determine which functions represent all parameter-dependent solutions best; the latter then span a reduced space. In the case of evolution equations or dynamical systems, reduced order models need to properly capture transient phenomena. In this direction, dynamical low-rank approximation (DLRA) methods have been developed (see, e.g., [5, 3]). They aim at computing an approximation in time-dependent reduced spaces that are through principles which are local in time (e.g., Dirac-Frenkel principle). Furthermore, probabilistic variants of MOR methods using randomized numerical linear algebra widely used in data science (see, e.g., [1, 2]) have recently emerged for improving computational performance of classical MOR methods. Here, one example is using a randomized singular value decomposition which allows one to deal with very large matrices in data science and in the case of MOR with large-scale simulations (see e.g., [4]). These randomized methods have also been used in [6] to construct approximation spaces that are (almost) optimal locally over time slabs. This is done by solving in parallel evolution problems with independent random initial times and initial conditions.

The aim of this PhD project is to develop new efficient MOR methods for parameter-dependent dynamical systems or evolution equations, by combining DLRA and randomized (parallel in time) methods. One possible application we will consider in this project will be full waveform inversion, where one tries to determine the property of the subsurface in a certain area to assess the risk for earthquakes. However, the methods are widely applicable e.g. also in medicine thus allowing us to target the applications to the student's interest.

Supervisors

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Funding and location

This project is funded by a joint PhD program between Centrale Nantes (France) and Stevens Institute of Technology (Hoboken, USA). The PhD candidate will spend two years at Stevens (Department of Mathematical Sciences) and two years in Nantes (Laboratoire de Mathématiques Jean Leray), with covered travel costs between institutions.

References

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