Branched covers in low dimensions
Example sheet 1

January 17, 2021

Solutions are accepted in English or French, and they are due on **January 24**.

You **can** work in groups, but solutions have to be **written up** and **submitted individually**.

If $i < j$, you can use the statement of problem i to solve problem j even if you haven't solved problem i. (Same for different parts within one problem, if there are more points in one problem, and you can solve later points even if you haven't solved earlier ones.)

Problems

1. (a) Let X be a finite simplicial complex (or a finite CW complex) with c_k simplices (cells) of dimension k for each k. Show that $\sum(-1)^k c_k = \chi(X) := \sum(-1)^k \dim H_k(X; \mathbb{F})$. (Here $\mathbb{F}$ is any field.) This justifies the fact that one can count simplices instead of homology ranks when computing Euler characteristics.

   (b) Show that if X is a d-fold cover of Y, and Y is a finite simplicial complex or a finite CW complex, then $\chi(X) = d \cdot \chi(Y)$.

2. (a) Let $p\colon X \to Y$ and $q\colon Y \to Z$ be branched covers between surfaces. Show that the composition $q \circ p\colon X \to Z$ is a branched cover.

   (b) Use point (a) and the branched covers we constructed in the lectures to show that there is branched cover from any closed (i.e. compact and without boundary) oriented surface to S^2, branched over three points. (Hint: the three points can be chosen as the north and south poles, and one point on the equator.)

3. Let $C \subset \mathbb{CP}^2$ be a non-singular curve of degree d, and $p \in \mathbb{CP}^2 \setminus C$ a point that does not belong to any inflection line of C. Show that there are exactly $d(d-1)$ complex lines passing through p that are tangent to C.