Branched covers in low dimensions
Example sheet 2

January 25, 2021

Solutions are accepted in English or French, and they are due on **January 25**. Please scan your solutions so that they're legible; pdf is the preferred format (there are apps to do that on your phone). They should be emailed to marco.golla(at)univ-nantes.fr.

You **can** work in groups, but solutions have to be **written up** and **submitted individually**.

If $i < j$, you can use the statement of problem i to solve problem j even if you haven't solved problem i. (Same for different parts within one problem, if there are more points in one problem, and you can solve later points even if you haven't solved earlier ones.)

Problems

1. Let $C \subset \mathbb{CP}^2$ be a non-singular curve of degree d, and $p \in \mathbb{CP}^2 \setminus C$ a generic point. Compute the number of lines passing through p that are tangent to C.

2. Give an example of a topological space (e.g. simplicial complex, CW complex, or manifold) P such that $H_2(P;\mathbb{Z}) = 0$ but $H_2(P,\mathbb{F}_2) \neq 0$, with proof. (Bonus: do it for $\mathbb{F}_p$ for all primes p.) Here $\mathbb{F}_p$ is the field with p elements.

3. (a) Describe all 3-fold cover of the 'eight' complex (i.e. the complex with one 0-cell and two 1-cells) and compute their monodromy representations.
   (b) Using point (a) or otherwise, show that there exists a 3-fold cover $S^2 \to S^2$ branched over three points.