# Branched covers in low dimensions
# Example sheet 4

February 8, 2021

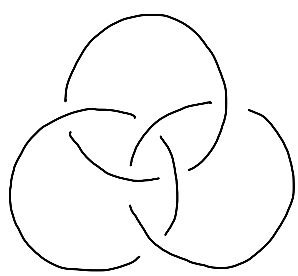
Solutions are accepted in English or French, and they are due on **February 15**. Please scan your solutions so that they're legible; pdf is the preferred format (there are apps to do that on your phone). They should be emailed to marco.golla(at)univ-nantes.fr.

You **can** work in groups, but solutions have to be **written up** and **submitted individually**.

If $i < j$, you can use the statement of problem $i$ to solve problem $j$ even if you haven't solved problem $i$. (Same for different parts within one problem, if there are more points in one problem, and you can solve later points even if you haven't solved earlier ones.)

## Problems

1. The *Borromean rings* form a 3-component link $B$ depicted in the figure below.
   (a) Show that the linking number between any two components of $B$ is 0.
   (b) Show that $B$ is not unlinked. (Hint: find an isotopy that looks like the diagram of the Whitehead link we showed in the lectures.)



2. (a) Exhibit a 4-fold (non-cyclic) branched cover $T^3 \to S^3$ and determine its branching locus and monodromy. (Hints: use the observation that $p \times p'$ is a branched covering if $p$ is a branched covering and $p'$ is an ordinary cover, and the double cover from the annulus to the 2-disc $A \to D^2$ we've seen in the lectures. More hints are available via email.)
   (b) Show that $T^3$ is not a double cover of $S^3$. (Hints: argue by contradiction. Show that if $M$ is a double cover of $S^3$ then the deck transformation group acts like multiplication by $-1$ on $H_1(M)$. Then show that $T^3$ cannot have an orientation-preserving map that acts in this way. You can use the fact that there are three classes $\alpha, \beta, \gamma \in H^1(T^3)$ such that $\alpha \cup \beta \cup \gamma \neq 0 \in H^3(T^3)$.)