Branched covers in low dimensions
Example sheet 5

February 15, 2021

Solutions are accepted in English or French, and they are due on **February 29**. Please scan your solutions so that they're legible; pdf is the preferred format (there are apps to do that on your phone). They should be emailed to marco.golla(at)univ-nantes.fr.

You **can** work in groups, but solutions have to be **written up** and **submitted individually**.

If $i < j$, you can use the statement of problem i to solve problem j even if you haven't solved problem i. (Same for different parts within one problem, if there are more points in one problem, and you can solve later points even if you haven't solved earlier ones.)

Problems

1. Let η_2 be the cobordism group of non-orientable surfaces (i.e. possibly non-orientable surfaces up to possibly non-orientable cobordisms, endowed with $\sqcup$ as an operation).

   (a) Show that $\mathbb{RP}^2$ does not bound a compact (non-orientable) 3-manifold.

   (b) Show that $\mathbb{RP}^2 \# \mathbb{RP}^2$ *does* bound a compact (non-orientable) 3-manifold.

   (c) Deduce that $\eta_2 \cong \mathbb{Z}/2\mathbb{Z}$, generated by $\mathbb{RP}^2$.

   (Hint: the first point is similar to one of the questions in an earlier problem set.)

2. Show that if M_1 and M_2 are two oriented closed n-manifolds, then $M_1 \sqcup M_2$ is cobordant to $M_1 \# M_2$.

3. Show that $\mathbb{CP}^2$ does not bound a closed, oriented 5-manifold. Deduce that $\Omega_4 \neq 0$. (Hint: this is similar to Problem 1 above.) Bonus: can you generalise this argument to some other Ω_n?