

Branched covers in low dimensions

Example sheet 6

March 1, 2021

Solutions are accepted in English or French, and they are due on **March 8**. Please scan your solutions so that they're legible; pdf is the preferred format (there are apps to do that on your phone). They should be emailed to marco.golla(at)univ-nantes.fr.

You **can** work in groups, but solutions have to be **written up** and **submitted individually**.

If $i < j$, you can use the statement of problem i to solve problem j even if you haven't solved problem i . (Same for different parts within one problem, if there are more points in one problem, and you can solve later points even if you haven't solved earlier ones.)

Problems

1. Suppose that $p: X \rightarrow Y$ is a d -fold cover between closed, orientable, smooth n -manifolds, branched over a collection F_1, \dots, F_m of embedded codimension-2 submanifolds in Y . Let c_i be the cardinality of $p^{-1}(x_i)$ for $x_i \in F_i$, for each $i = 1, \dots, m$. Show that:

$$\chi(X) = d\chi(Y) + \sum_{i=1}^m (c_i - d)\chi(F_i)$$

2. Let $F \subset X$ is a closed oriented surface and N be a tubular neighbourhood of F .
 - (a) Show that $H_2(N, \partial N) \cong \mathbb{Z}$, generated by a meridional disc of F .
 - (b) Show that the map $\mathbb{Z} \cong H_2(F) \cong H_2(N) \rightarrow H_2(N, \partial N) \cong \mathbb{Z}$ is multiplication by $F \cdot F$.
3. If $F_1 \cup \dots \cup F_m \subset Y$ is the branching locus of a cyclic d -fold cover $p: X \rightarrow Y$ and F_j is orientable for each j , then:
 - (a) $F_j \cdot F_j$ is divisible by d for each j ;
 - (b) $\sum_j F_j \cdot F_j$ is divisible by d^2 .