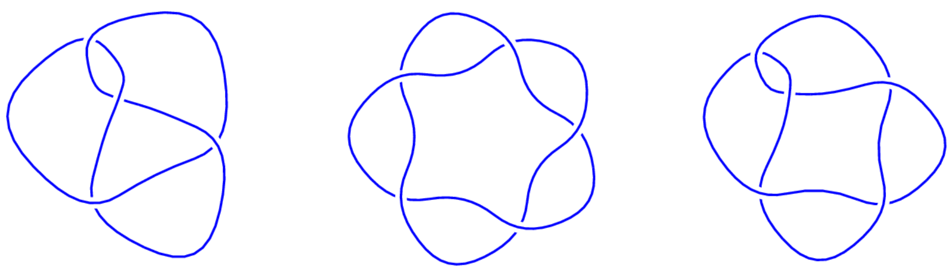
Introduction to knot theory
Example sheet 3

March 11, 2019

These exercises can be done in groups or individually; please refrain from looking/asking for solution online. Upon request, hints might be given. Homework can be handed in in English, French, or Italian; the deadline is **Mar 18, 2019**.

1. Compute the Jones polynomial and Arf invariant of the figure-8 knot (4_1) and of the knots 5_1 and 5_2, below. Use the state-sum expression and the skein method at least once. Pictures are taken from `http://www.indiana.edu/~knotinfo`.



2. (i) Show that $|V_L(-1)| = |\Delta_L(-1)|$. (This number is called the *determinant* of the link. Why?) (ii) Show that if L is an ℓ-component link, then $V_L(1) = (-2)^{\ell-1}$.

3. Recall that the braid group on n strands is the group generated by elements $\sigma_1, \dots, \sigma_{n-1}$, where

$$\sigma_i = |\cdots| \times |\cdots|, \qquad \sigma_i^{-1} = |\cdots| \times |\cdots|$$

with $i-1$ strands to the left of the crossings, and $n-i-1$ to the right. A *positive braid* is a braid that is a composition (i.e. stacking) of σ_i's (with their orientation). Let K be the closure of a positive braid β; prove that K is the unknot if and only if σ_i appears in β exactly once for each i.

4. Prove that any mutant of the unknot is itself unknotted.